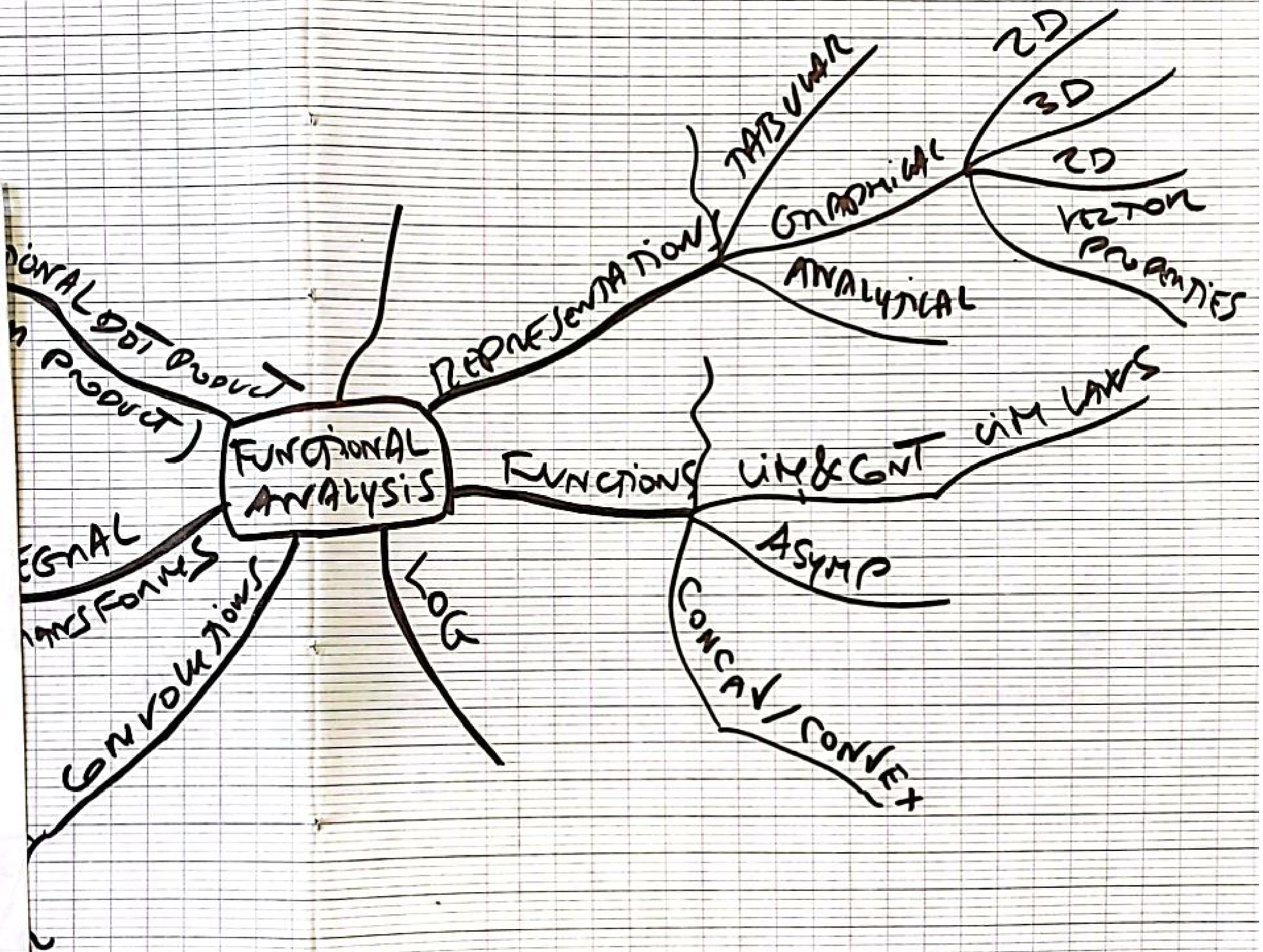
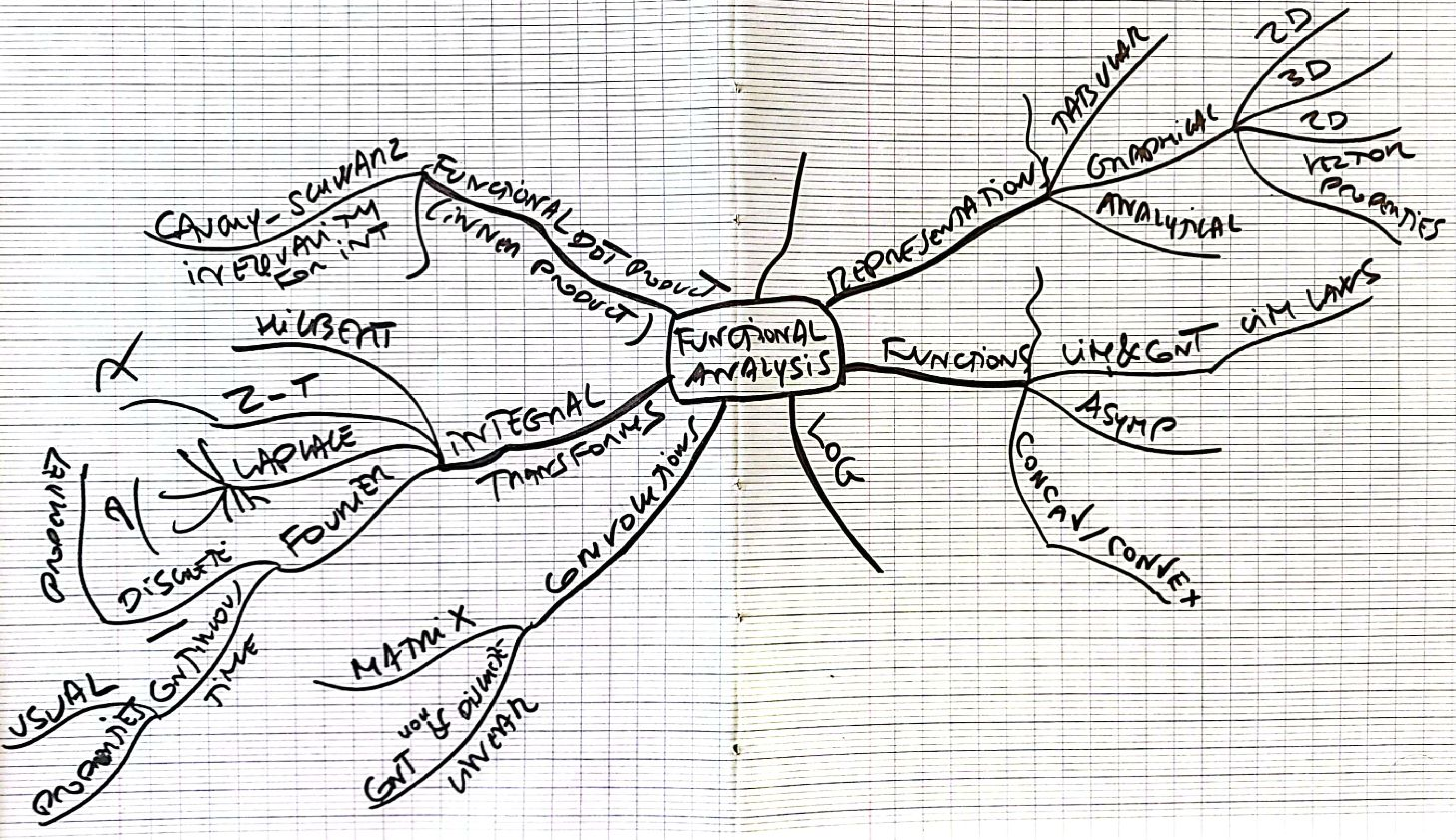


isoz FUNCTIONAL ANALYSIS


FUNCTIONAL ANALYSIS
REPRESENTATION
TABULAR
GRAPHICAL
2D
3D
2D
VECTOR PROPERTIES
ANALYTICAL
FUNCTIONS
LIM & CONT
LIM LAWS
ASYMP
CONCAV / CONVEX
LOG
FUNCTIONAL DOT PRODUCT
(INNER PRODUCT)
CAUCHY-SCHWARZ
INEQUALITY FOR INT
HILBERT
Z-T
LAPLACE
FOURIER
INTEGRAL
TRANSFORMS
DISCRETE
CONTINUOUS TIME
PROPERTIES
USUAL
PROPERTIES
CONVOLUTIONS
MATRIX
CONT & DISCRETE
LINEAR

FUNCTIONAL ANALYSIS

- function spaces

roots → study of transformations (Fourier)

↘ diff eqn & integrals

- why analysis: import case of f^{ns}?

variation phenom correlated with ___ math or several SM var

Study of variations tools

- "laws"
- no usage of graphical rep nor access to exper measures

R f

- + linked $\vec{v}$ calculus

Representations

1 rep ±-values by tables, dims

2 how math analyse the properties of these rep^{n} ← abstract tools

D

f univalent = unary if

nb of its arguments (param or var) = 1

f 2 ary bivalent

f n-ary

$f : A_1 \times A_2 \times \ldots \times A_n \longrightarrow B$

Tabular

values of indep var

x	$y = f(x)$
x_1	$y_1 = f(x_1)$
⋮	
x_n	$y_n = f(x_n)$

corr. values

transform ‖ var of f y_1, y_2, y_3

~~f args~~ $f(a_1, a_2, \ldots, a_n)$

arguments of f

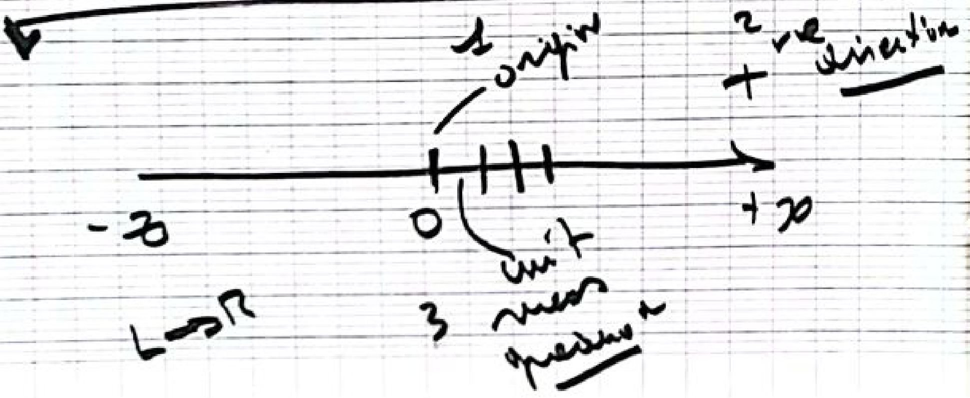
f^{d} dependence betw measured phys qt^{ies}

- Can be generalized to any multivar f regardless its def domain
- ∴ permit see ~~be~~ directly behavior of f

∴ simple & attractive visual analysis of its quant properties

GRAPHICAL

- not rel real v purely i nbs can all be rep^ed as simply by points on a numerical x axis



letter
R ⊙ func^n rep ⊙
physics ⊙ of pure positional at the loc^n of centroid of system

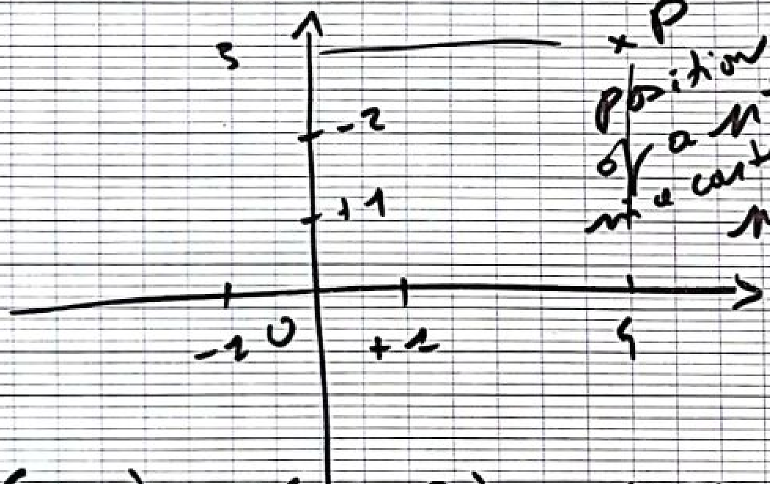
- • every nb
∴ 2 substant real nbs

- ∀ nbs on ∀ points

ℂ - nb at each pt of axis
→ each nb rep a • on ~~the~~ a unique graduation & back to each pt on grad is a single nb which is the image

2D plane

f's of one var (but also pts!)



position of a pt on the cartesian plane

$P(x,y) = P(+4,+3) = P(4,3)$

cart coordinates

graph consists of points

$(x,y) = (x, f(x))$

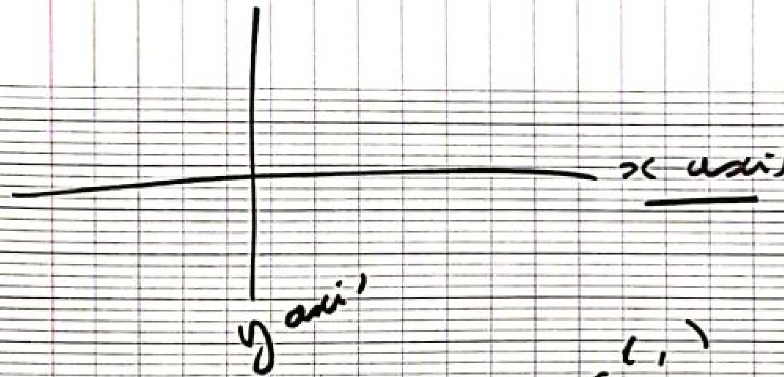
Euclidean space

ordered pairs

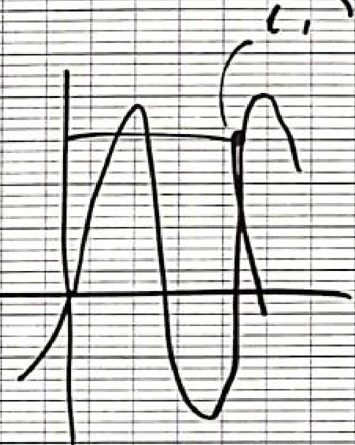
(a,b)

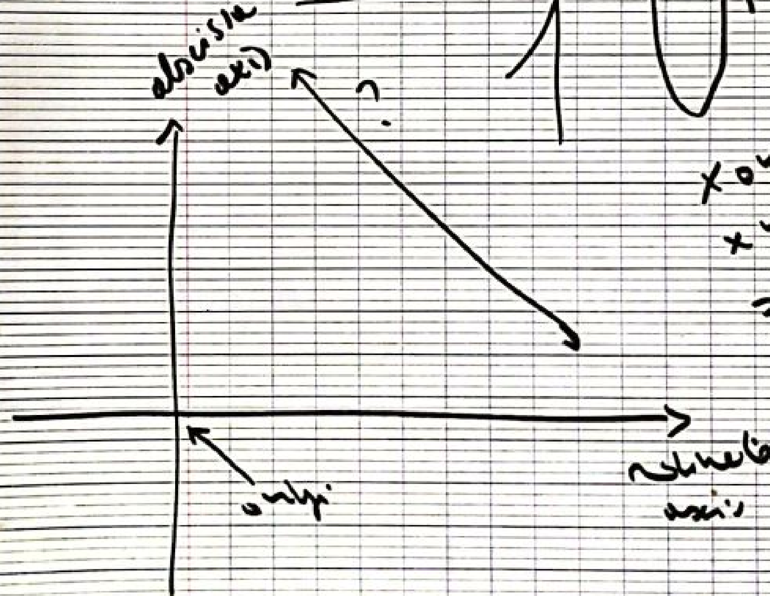
$\mathbb{R}^2$ — set of all pts of the plane

rules of eucl geom



$y = f(x)$





xOy

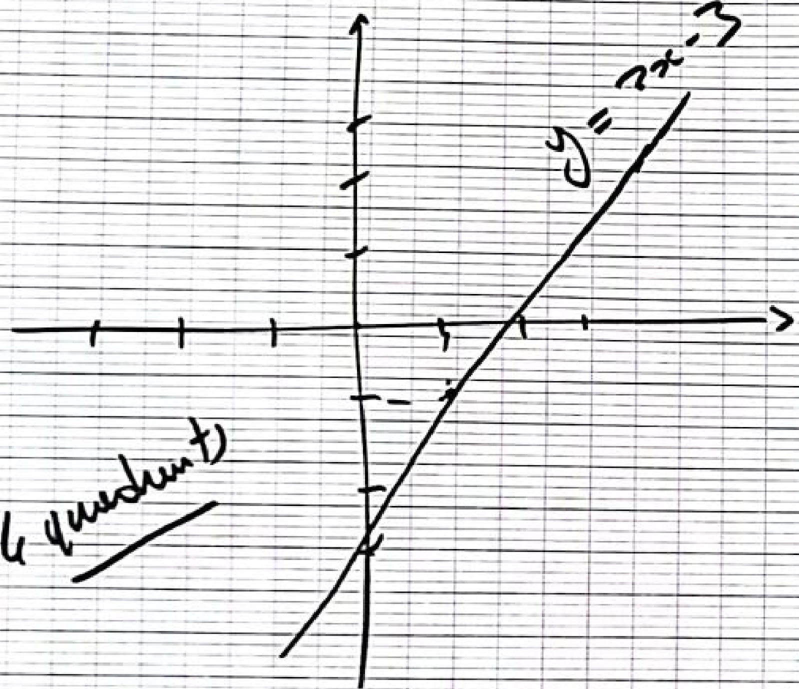
xy

xOy

Oxy

$y = f(x) = 2x - 3$

$x \in [-3.3, +3.3]$

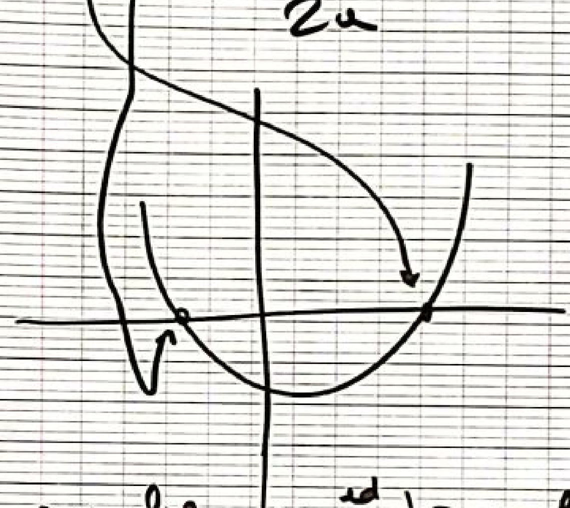


4 quadrants

x	$2x-3$	Point $(x, 2x-3)$
-1	$2\cdot(-1)-3=-5$	$(-1,-5)$
0		
$+1$		
$+3$		

$y = f(x) = ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



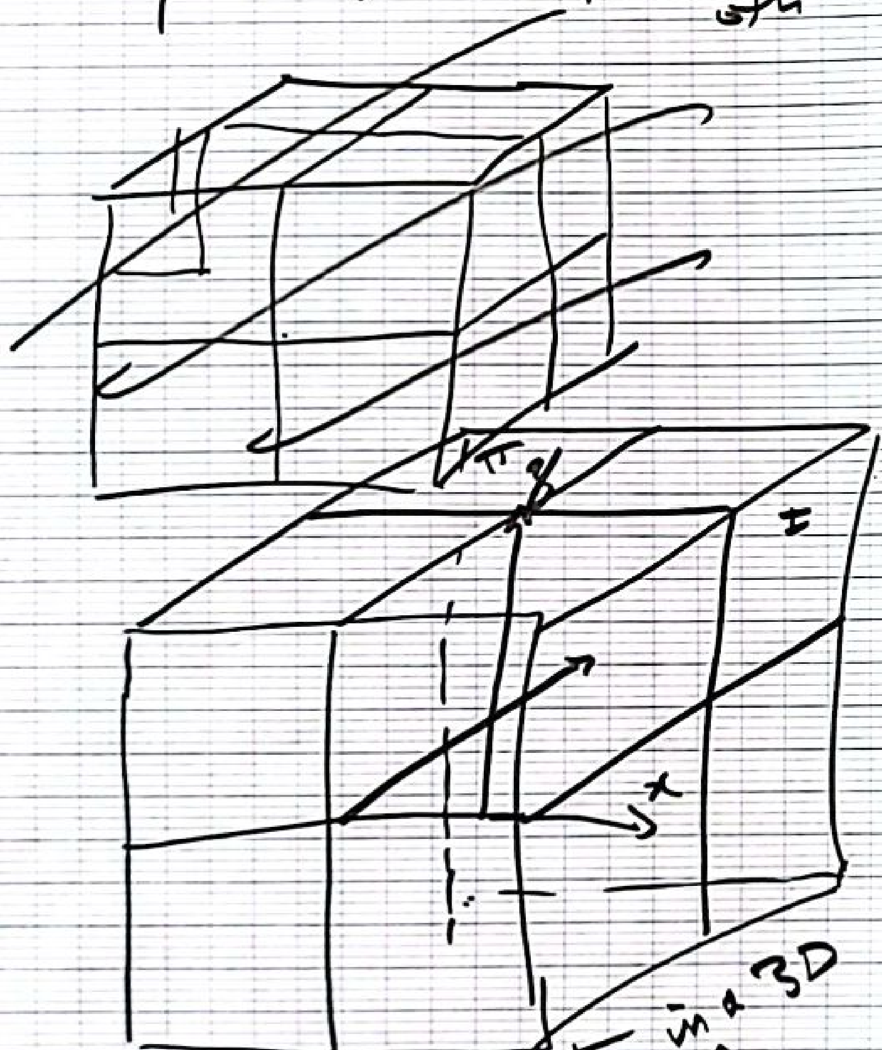
can be gen^ed to polyn 3^rd 4^th 5^th degree

(Galois G → not poss to get a general alg exp of roots of pol eqn of 5th degree & higher)

OPEN HIGH LOW CLOSE

3D

trivial f   a parameter dep. on two oth

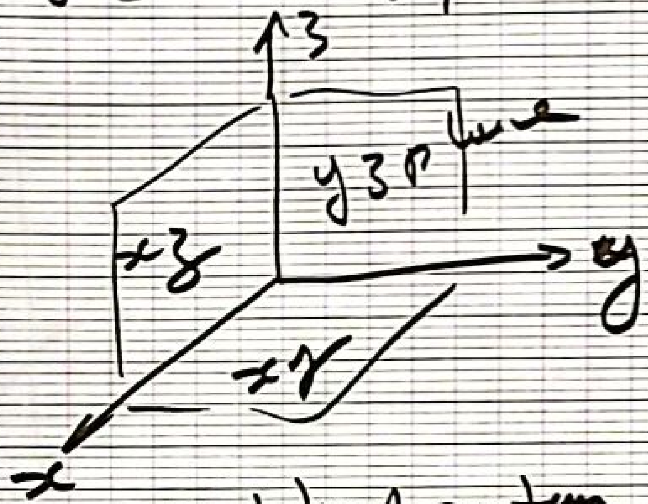


I → VIII

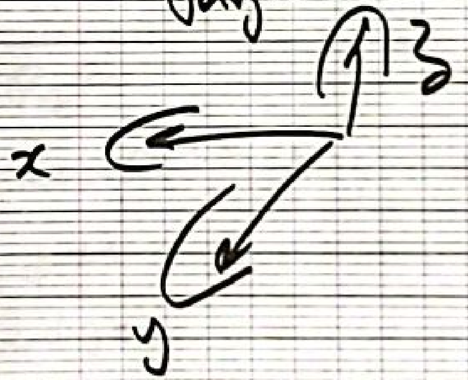
quadrants in a 3D orthogonal system

~~x y~~ x, y, z

$$(x, y, z) = (x, y, f(x, y))$$



Right hand system



$f : \mathbb{R} \times \mathbb{R} \longmapsto \mathbb{R}$

$z = f(x,y) = \dfrac{12x}{1+x^2+y^2}$

$(x,y) \in [-10,+10] \times [-5,+5]$

Maple 6.00

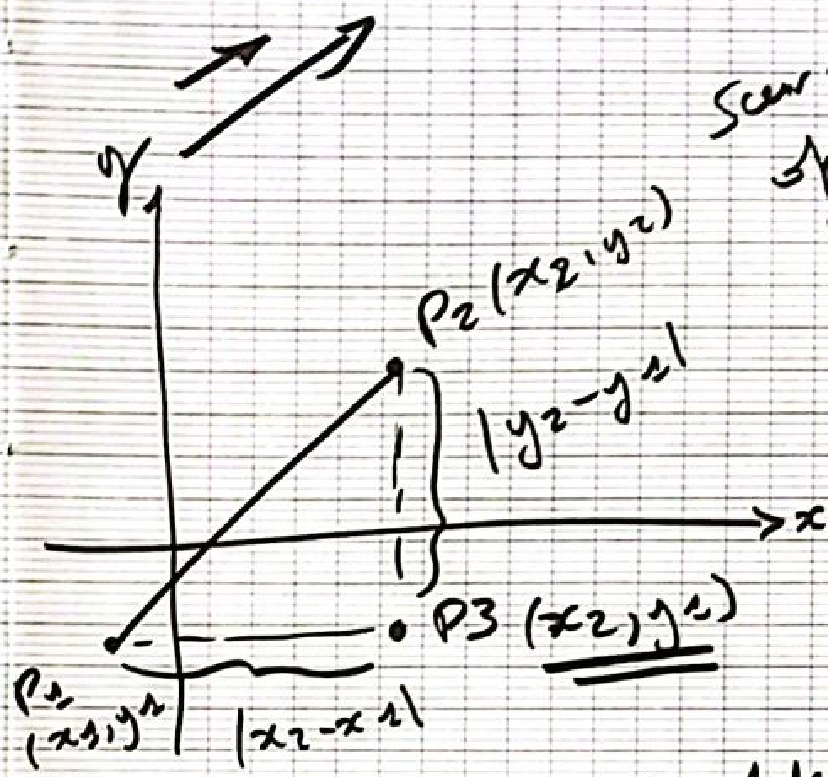
(Contour lines = isoline isoquant
same height
MATLAB

2D Vector ~~...~~ Representation

- analytical geom
- thus ~~...~~ ~~...~~ norm plotting software
& applying Pythagorean T
- Main idea of a planar vector

$P_1\ (x_1, y_1)$
$P_2\ (x_2, y_2)$

Scenario of 3 points in a plane



$\overline{P_1P_2}$ is a $\vec{v}$ but not translated ~~at~~ at the ~~...~~ origin of ref

$x_1 \neq x_2$
$y_1 \neq y_2$

$$[d(P_1,P_2)]^2 = \underbrace{[d(P_1,P_3)]^2}_{|x_2-x_1|} + \underbrace{[d(P_3,P_2)]^2}_{|y_2-y_1|}$$

Since $\forall x \in \mathbb{R}$ $|x|^2 = x^2$

$$d(P_1, P_2) = \sqrt{[d(P_1,P_3)]^2 + [d(P_3,P_2)]^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If $x_1 = y_1 = 0$

We end up with a R named

norm = module = distance

when origin of $\vec{v}$ is translated

on the ~~of~~ origin of the ref

$$d(P_1, P_2) = \sqrt{x_2^2 + y_2^2} = \|\vec{v}\|$$

T $P_1(x_1, y_1)$ $P_2(x_2, y_2)$

$P_3(x_3, y_3)$ on mediator

$$d(P_1, P_3) = d(P_2, P_3)$$

PF

"analytical expression"

D Any f of the form of polyn

of degree 1 with ~~coef~~ constant

real coef

$$y = f(x) = ax + b$$

is the analytical expression

* straight line linear equation

of slope a & intercept b

when $x = 0$

If : $y = f(x) = b$

$x = f(y) = a$

Properties of visual rep^{ns}

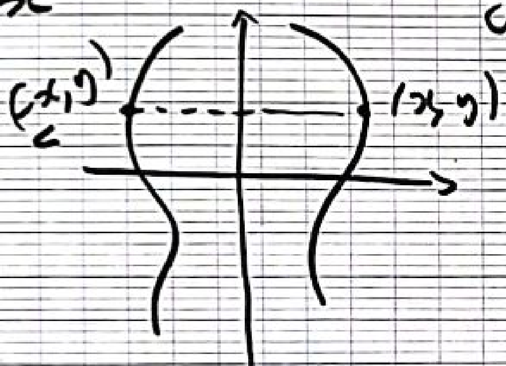
univariate functions

$y = f(x)$

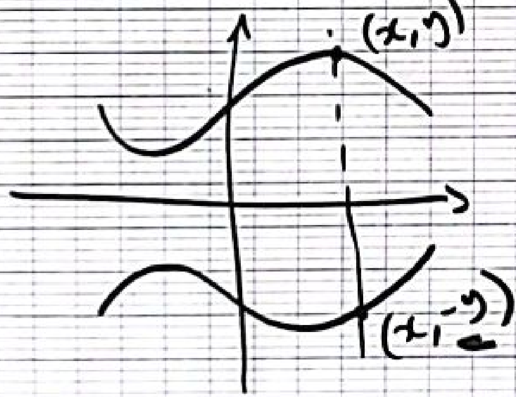
P1 ~~graph~~ graph of symmetrical about the y-axis

$x \longrightarrow -x$

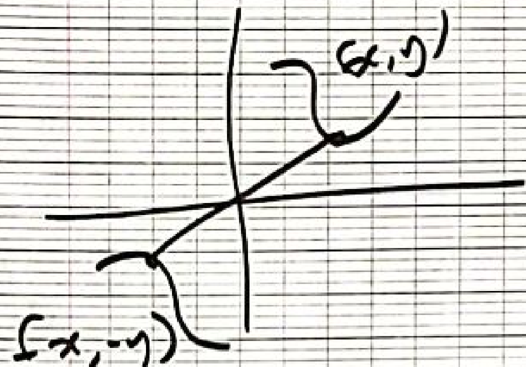
$y =$



P2 — x

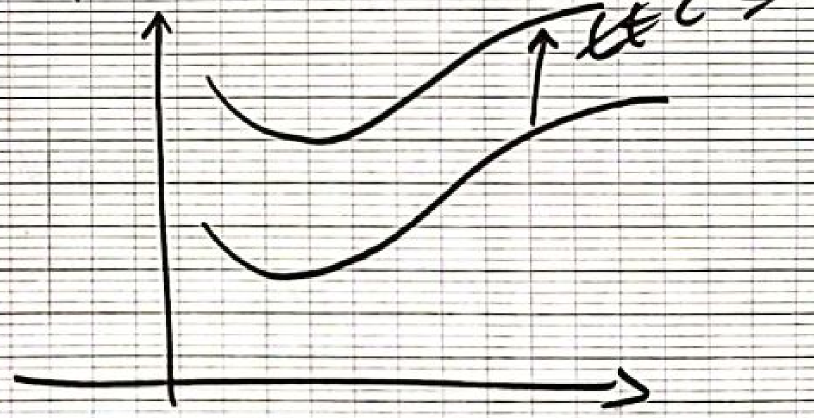


P3 origin

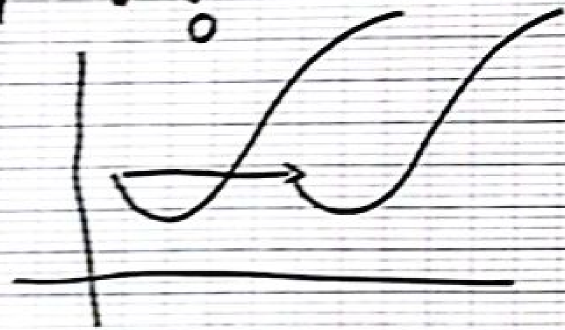


P4 $y = f(x) + \cancel{cte}\, c^{te} \geq 0$

shifted translated vert^{ly} upwards of a distance



$y = f(x - c^+)$ $\neq 0$ ↓ NB

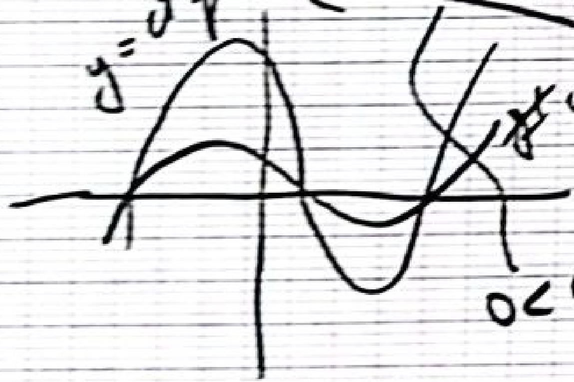


\+

stretch or compress vertically

$c^+ > 0$ $\quad 0 < c^+ < 1$

$y = c^+ f(x)$ $\quad c^+ > 1$

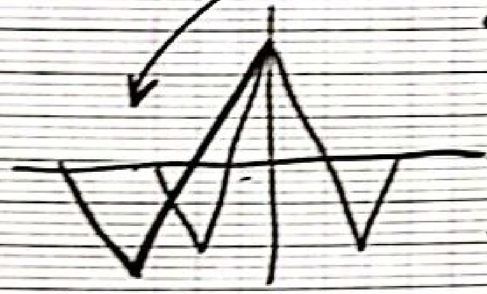


$y = f(x)\, c^+$

$0 < c^+ < 1$

hor ↓ $y = f(c^+ x)$

$0 \le c^+ < 1$



hor stretch

N Translate, stretch, compress or apply a sym is transforming it plot → "transformed"

D_1 C^+ on I for each pair (x_1, x_2) $\forall$

$f(x_1) = f(x_2)$

$\forall (x,y) \in I^2 : f(x) = f(y) = C^+$

D_2 ↗ $\forall (x,y) \in I^2 \wedge (x \le y) :$

$f(x) \le f(y)$

↘ $\geq$

D4 strictly ↗

$\forall (x,y) \in I^2 \wedge (x<y) : f(x) < f(y)$

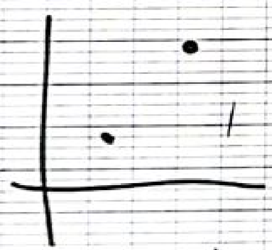
D5 ↘

Analytical ⇐

- analytic expression = closed form
- function y of x is given analytically

$f(x) = y = ax + b$

linear eqⁿ = affine f



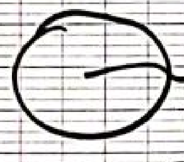
$P_1(x_1, y_1)$

$P_2(x_2, y_2)$

Slope = ratio $\dfrac{\text{vertical growth}}{\text{horiz}}$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

// if same slope



$C(h, k)$

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

D Natural Dom of def given by an analyt expression is the set of x values for which the expⁿ on the right hand side has a definite value

$$y = \frac{x+1}{x-1}$$

$x = 1$ singularity

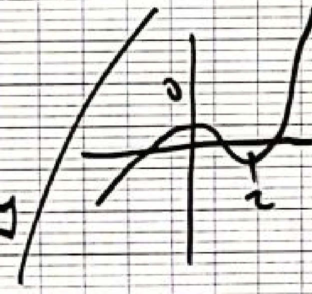


Table of variations

$x^3 - 3x^2 + 2$

x	$-\infty$		0		2		$+\infty$
f'		+	0	−	0	+	
f		↗		↘		↗	

Functions

- set inputs $\mathcal{R}$ set permissible outputs

each • —— f —— • each

exactly

- formula algorithm

  picture graph

  table

  implicitly as the inverse to another f

  or sol^n of a dif^{al} eq^n

D

strict

Complementary

D1

$D \xrightarrow{f} E$

$f : D \to E$

$\boxed{f : D \mapsto E}$

$y = f(x)$

dep var

indep var

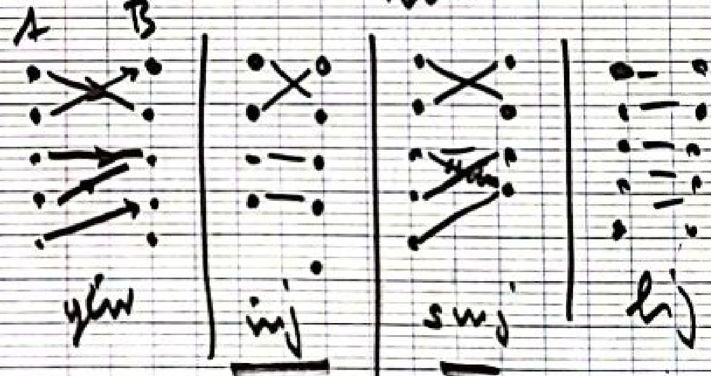
input

arg

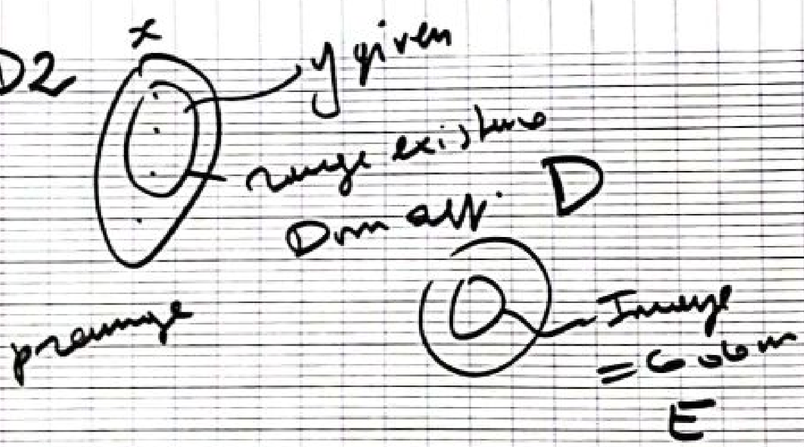
f^{al} dependency

~~yf~~ $y = y(x)$

value & symbol



D2

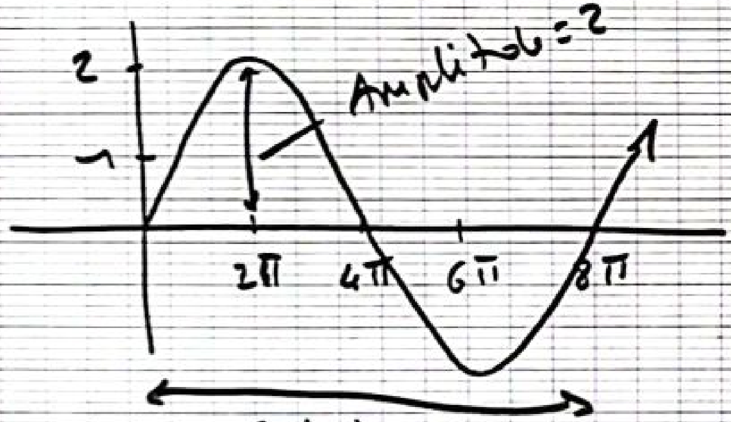


D3 periodic

$y = f(x \pm c^{te}) = f(x)$

smallest = period T

$y = 2\sin(\frac{1}{4}x)$



Period = 8π

Σ periodic with $\neq$ Ts Not nec. periodic!

There is no gen formula
to get period of a f
that is the Σ of oth fs

D4 Dif quot

$\frac{f(x+h)-f(x)}{h} =$ Growth quotient

x_0

D5 $f(x) = f(-x)$ even

odd $f(-x) = -f(-x)$

T x 2 even = even
x 2 odd = even
even x odd = odd

D6 $f+g$ $(f+g)(x) = f(x)+g(x)$

$-$ $-$ $-$

$\cdot$ $\cdot$ $\cdot$

$\div$

Def Dom $f+g$ $f-g$ $f\cdot g$

$\cap$

/ such $g(x) \neq 0$

D7 $y = f(u)$
$u = g(x)$

$y = f(g(x))$
$= (f \circ g)(x)$

Dom Def either = entire D
or part

If u ~~dep~~ on another var (n it is not itself a composite f)

$f(x)$ elementary

• linear

$f(x) = ax$
~~ato~~ slope

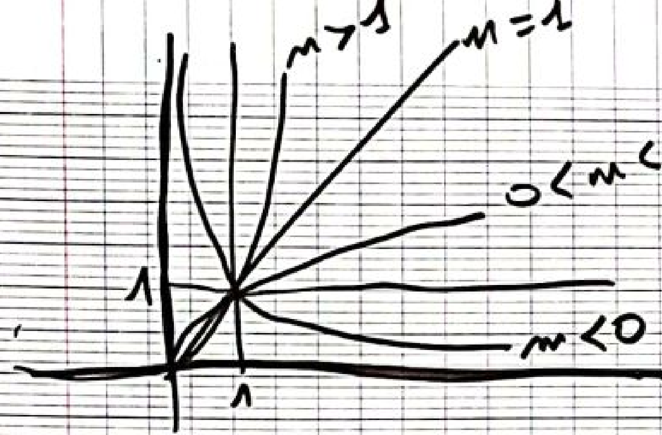
• affine

$= ax + b$

• power

$= x^{m \in \mathbb{R}}$

involving roots often named radical



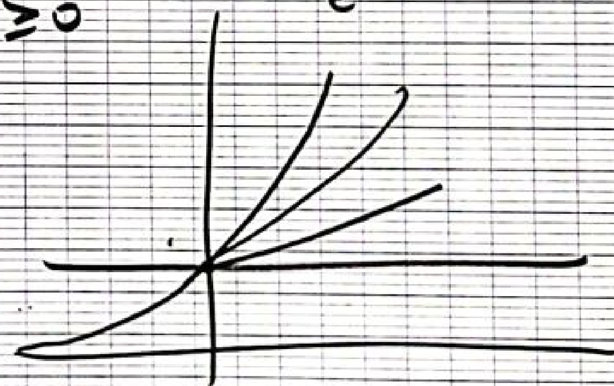
• Absolute value

$f(x) = |x|$

• exponential

$= a^x$
$e \in \mathbb{R}$

$a \geq 0$

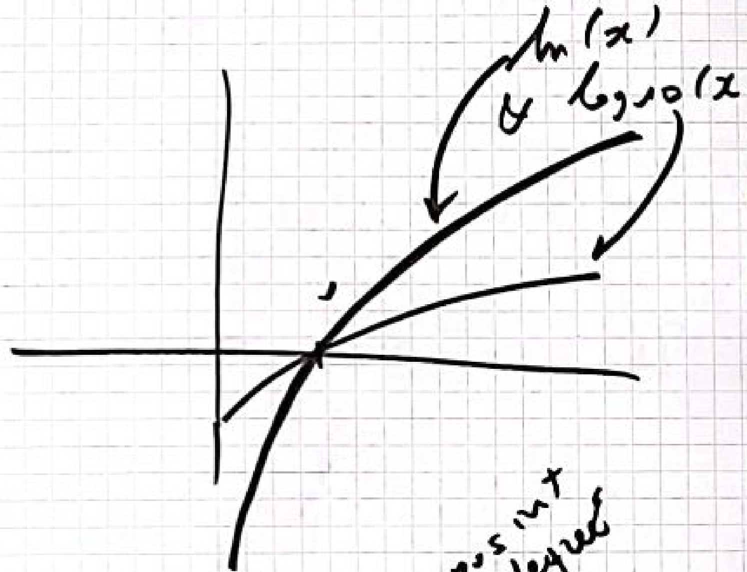


$a < 0$ not defined

Instead for $(-1)^{(0.5)} = \{i, -i\}$
$\mathbb{R} \longmapsto \mathbb{C}^2$

• log $= \log_a(x)$, $a \in \mathbb{R}^+$

$\mathbb{R}^+ \longmapsto \mathbb{R}$



• Per trig $\quad y = \sin(x)$

• polyn $= P(x) = \underbrace{a_n}_{\text{coef}} x^{\overset{\text{pos int degree}}{n}} + a_{n-1} x^{n-1} + a_0$

fonctions — logarithme

• Rat $\quad \dfrac{P(x)}{Q(x)}$

$$f(x) = \frac{x^3 - 2x}{2(x^2-5)}$$

not def at $x^2 = 5 \iff x = \pm\sqrt{5}$

$$\frac{x^2+2}{x^2+1}$$

def for all real

not all complex

since if x were a square root of -1

$\to 0$

• Algebraic result $+ - \times \div$

if we put to an int or not int power

$\therefore$ not $\nearrow$

• piecewise $\quad$ régularité par morceaux

$$f(x) = |x| : \begin{cases} -x & x < 0 \\ +x & x \geq 0 \end{cases}$$

- <u>Step f</u>

$f : [a,b] \in$ is step iff there exists a subdivision $(a_i)_{0 \le i \le n}$ of $[a,b]$

Such that $a_0 = a$ & $a_n = b$ &

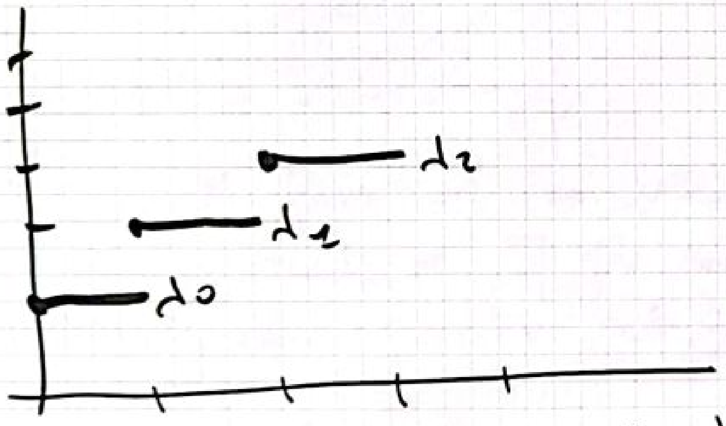
$(\lambda_0, \ldots \lambda_n) \in \mathbb{R}^n$

Such as :

$\forall \in \{0, \ldots, n-1\}, \forall x \in]a_i, a_{i+1}[$

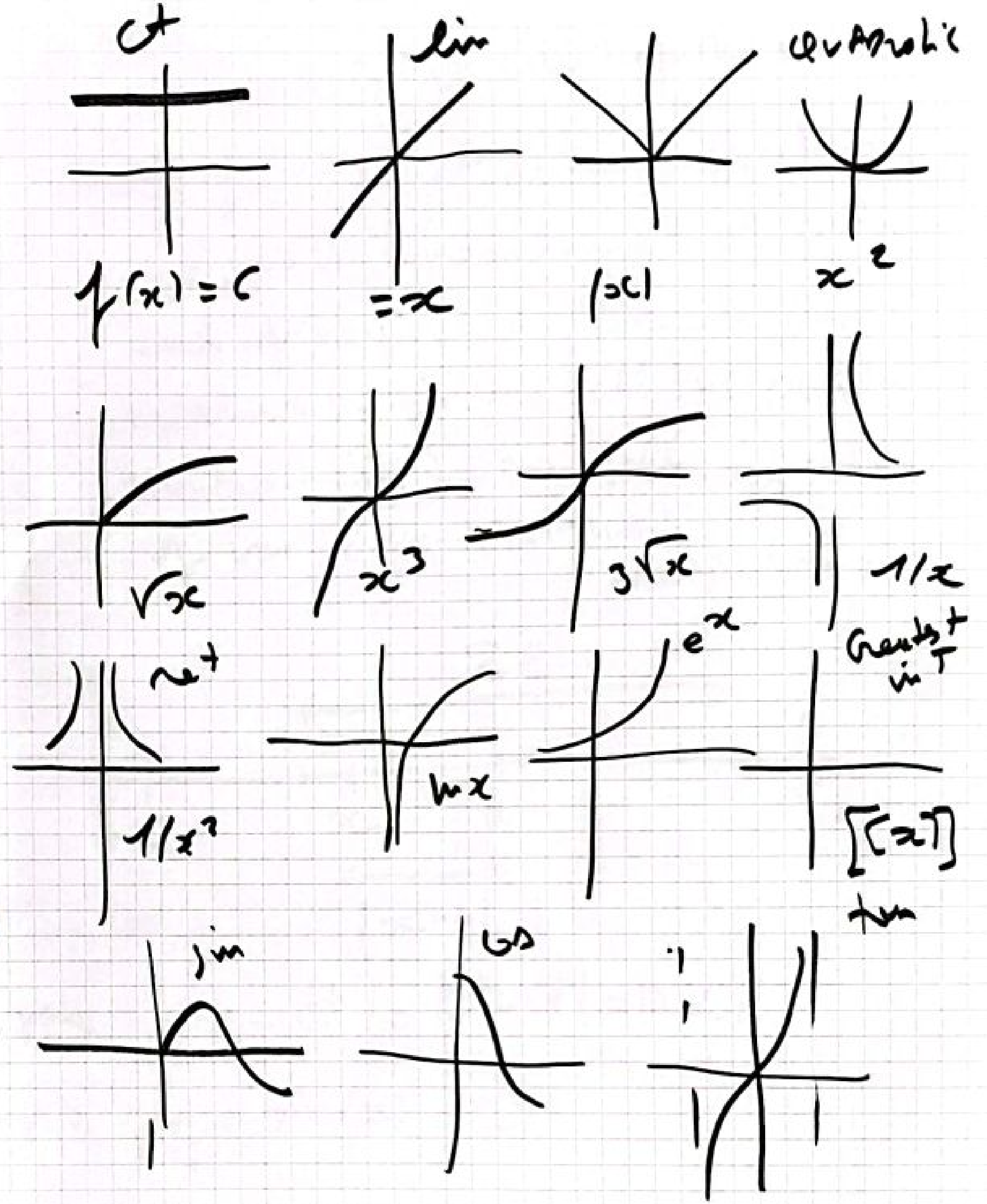
$\Leftrightarrow$

$f(x) = \lambda_i$



linear comb of all elementary fs

Dirac [illegible]



Lims & Cont

ordered var special type
which we define by the $\mathbb{R}$
"the var tends to a limit"

concept lim of a var

D

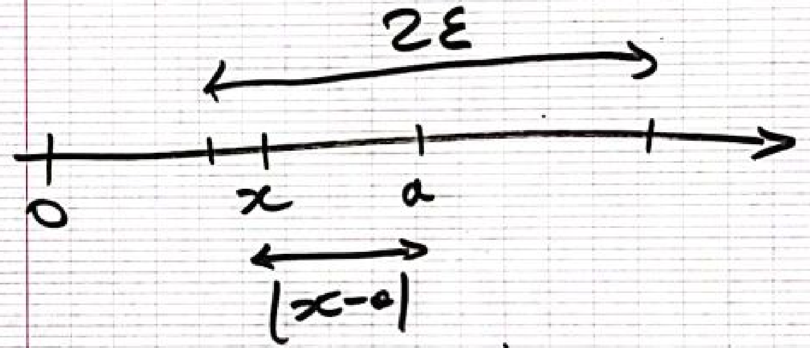
~~a is named limit of~~

a the limit of variable magnitude x, if for any arbitrarily small positive nb ε:

$$|x-a|<\varepsilon$$

If the nb a is the lim of the variable x, we say
x tends to the lim a

ct nb a is the limit of the var x
if for any given neighborhood, no matter how small of center a
and of radius ε, we can find value x such as all the points
corresponding to the following values of var belong to this neighborhood.



n1 $|x - c^{+}| = |c^{+} - c^{+}| = 0 < \varepsilon$

n2 $y = \sin(x)$ ≠ ?

D $|x| \to \infty$, $\forall M > 0$ $\cancel{\exists}$ $|x| > M$

- $x \to +\infty$ $M < x$

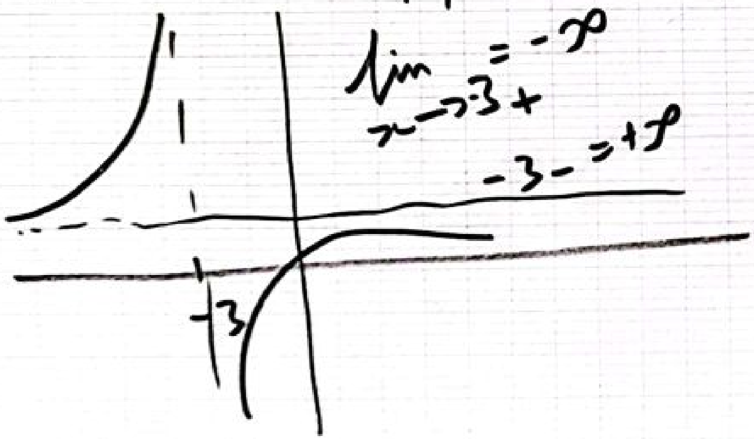
$x \to +\infty$, $\forall M > 0 : x > M$

$-\infty < x < M$

D $\cancel{\exists}$ $y = f(x)$ neigh.. of a

$\lim_{x \to a} = b \iff \forall x \in E, \underline{\exists a, b}, \forall \delta > 0\ \forall \varepsilon > 0 :$

$|x - a| < \delta \Rightarrow |f(x) - b| < \varepsilon$



$\lim_{x \to 3+} = -\infty$

$-3- = +\infty$

E1 $\lim_{x \to +\infty} \frac{x+1}{x} = 1$

we have to ~~pro~~ prove that for any small ε the inequality

$|1 + \frac{1}{x} - 1| < \varepsilon$ will be

satisfied as soon as $|x| > M$ where M is defined by the choice of ε

$|\frac{1}{x}| < \varepsilon$

$|x| > \frac{1}{\varepsilon} = M$

D $f(x)$ $E \subseteq \mathbb{R}$ $x_0 \in E$

cont on x_0 iff

$\lim_{x \to a} \cancel{\exists} f(x) = f(x_0)$

$\forall x_0 \in E\ \forall \varepsilon > 0\ \exists \delta > 0\ \forall x \in E :$

$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$

R1 cvt to L
$x > x_0$
R2 with cvt inV $f^{\wedge}$ = homeomorph
R3 oscill^n

C $f(x)$ cvt on x_0 iff $f(x)$ cvt L

Limit laws

Asymptots

~~Df~~
Concavity / Convexity