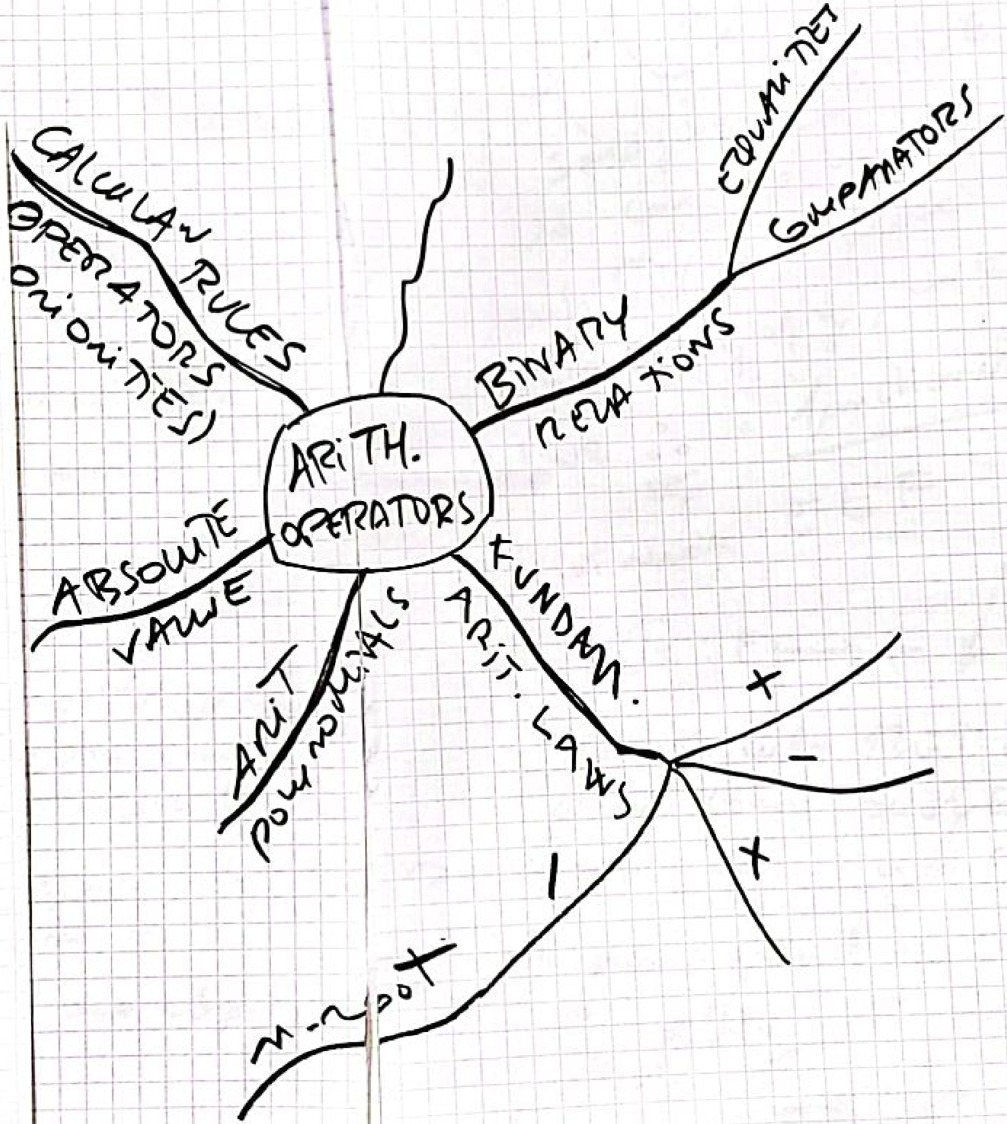
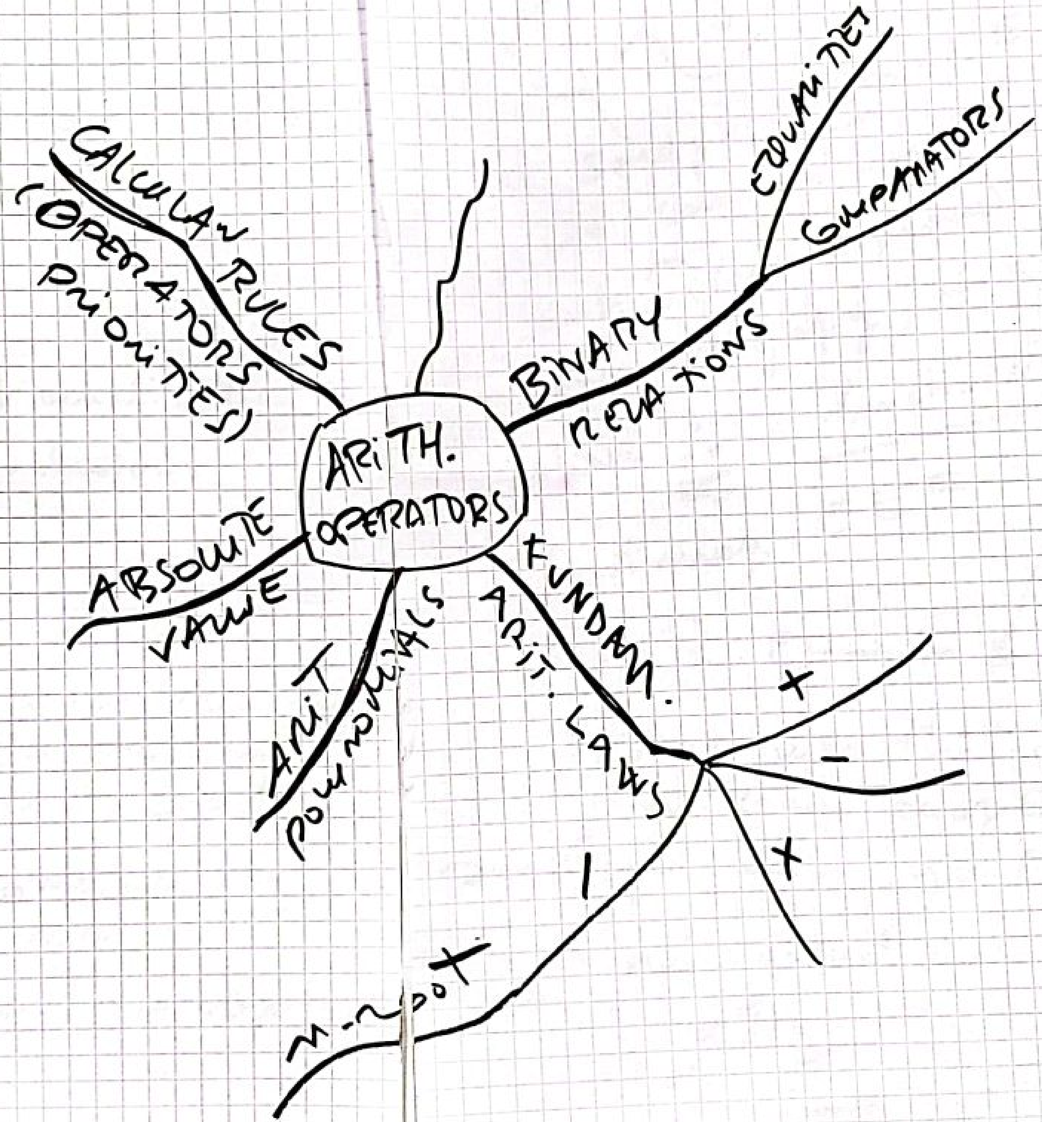
ISO2

ARITHMETIC OPERATORS





Scanné avec CamScanner

ARITH OPERATORS

- $+ \quad - \longrightarrow \times \quad \div \quad /$

"notional −"

could be enough → only +ve of −ve ~~nb~~ nbs

- Binary operators (relations)

$= \quad \neq \quad > \quad < \quad \geq \quad \leq$

↳ & /r ~~compare~~ ~~note~~ order of amplitude of elts → some conclusions

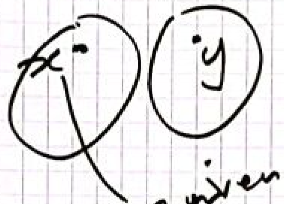
(Viète) 16TH

BINARY R

study – by observa^n & deduc^n (reasoning), calcula^n & comparison – configurations or ~~of~~ abstract / concrete R of its objects (nbs, forms, str) by seeking to establish log, num, or conceptual link between these objects

Ds ~~empty~~ sets

D1 E F not nec^y identical



if some given elts of E we can ass with a precise math rule R (unambiguous) one elt y of y, we define $\stackrel{\text{def}}{=}$ a "functional R" E to F that map

$R : E \longmapsto F$

math rule that associate to given any x of E

some — — y of F

So more general context

if xRy

"precedent" "preimage" of y

image of x through R

the Set of pairs (x, y) such ~~as~~ that

$x R y$ is a T stat generates "a graph"

or "representation" of R

we can rep these couples in a proper chosen way to make a graphical rep of R

. $R : f(x) = y$ of

D2 Consider non empty E

if

associated only/union tools to compare between them

"binary R" "comparison"

$x R y$

any elt x only of E

$\mathbb{N}\ \mathbb{Z}\ \mathbb{Q}\ \mathbb{R}$

for 4 line

were congruence rep^ed by lines in plane

whose points are given by

constraint of congruence

$= \neq > < \geq \leq$

<u>Equalities</u>

extensionality axiom of set theory

D1 2 elts are "=" iff $\equiv$

they have the same values

If $a = b$ and c given vector not

$\star$ any operation

then $a \star c = b \star c$

used solve or simplify

eliminate

LHS RHS

obviously (reflexivity)

$a = b \Leftrightarrow b = a$

trans

$$\left.\begin{array}{l} a = c \\ b = c \end{array}\right\} a = b$$

D2 if not strictly $=$ $=$ "inequal"

If $a > b$ or $a < b$

Then $a \neq b$

Scanné avec CamScanner

$\cong$: Congruence   approx^n

$\approx$ approx^n

$\equiv$ 2 elts are equiv $=$

$:=$ one elt is by def equal to another one

$\doteq$ " equal by def"

$\sim$ "follows the law = asymp equal

Comparators — and order any pair of nbs (also sets)

Fundamental

True for any $a, b, c \in \mathbb{R}$
$a \geq b$
$\leq b$

$\mathbb{R}$ tot^y ordered Group

D $\leq$ "relation order" $\geq$

Strict

$a < b$ and $b > c \Rightarrow a < c$

and: $a > b$ and $b > c \Rightarrow a > c$

$a > b$ and $b = c \Rightarrow a > c$

$a < b$ and $b = c \Rightarrow a < c$

if: $a < b$ and $c > 0 \Rightarrow ac < bc$

$a < b \Rightarrow a + c < b + c$ and

$a < b \Rightarrow a - c < b - c$

and vice versa

$0 < a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

$b < a < 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$

$a > b$ and $c > 0 \Rightarrow ac < bc$

$a < b$ and $c < 0 \Rightarrow ac > bc$

$0 < a < b$ and $n \in \mathbb{R}^{+*} \Rightarrow a^n < b^n$

$b < a < 0$ and $n \in \mathbb{N}^*$ even

$0 < a^n < b^n$

odd $a^n > b^n$

Finally

$0<a<b$ and $n \in \mathbb{N}^* \Rightarrow \sqrt[n]{a} < \sqrt[n]{b}$

$> \; < \; \leq \; \geq \; \gg \; \ll$

D $\underline{\mathcal{R} \subseteq A \times A}$

bin $\mathcal{R}$ generates a subset by the constraint it imposes on the elts of A (st satisfying the $\mathcal{R}$) i.e. the the property of bin:

P1 "Reflexive" if $\forall x \in A$ EMST ONAT (R)

$x \mathcal{R} x$

"symmetrical" (S) $x\mathcal{R}y \Rightarrow y\mathcal{R}x$

"anti ——" $(x\mathcal{R}y \text{ and } y\mathcal{R}x) \Rightarrow x=y$ (A)

"transitive" $(x\mathcal{R}y \text{ and } y\mathcal{R}z) \Rightarrow x\mathcal{R}z$ (T)

"convex" $x\mathcal{R}y$ or $y\mathcal{R}x$ (C)

D1 Strict order (T) $\rightarrow$ nec^y (A)

preorder (R)(T)

equivalence (R)(S)(T) EMST

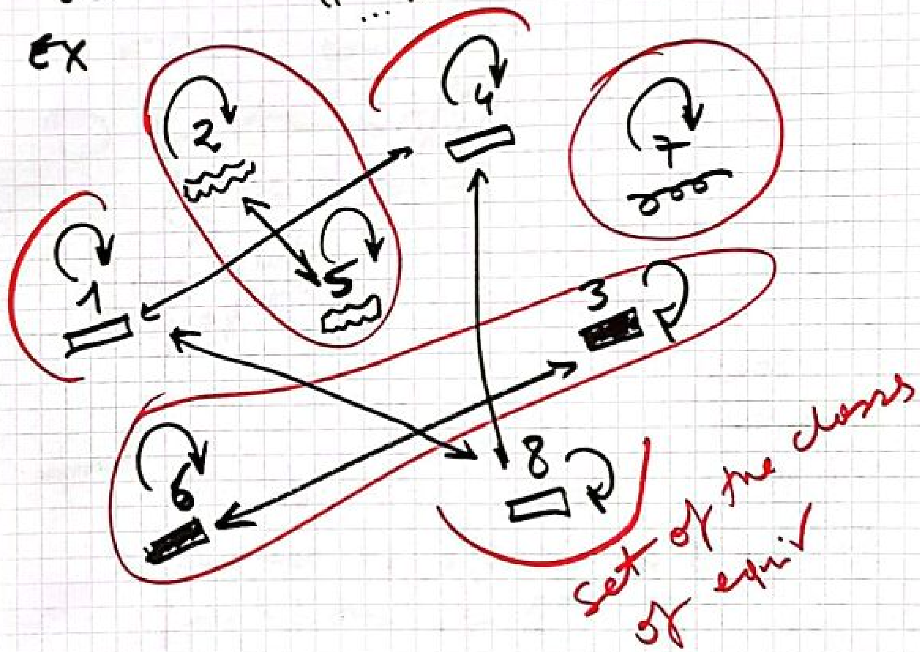
order (R)(T)(A) ONAT

total order (R)(T)(C)(A)

no natural order on $\mathbb{C}$

"... has the same ISBN..."

Ex



	$\neq$	$\neq$	$>$	$<$	$\leq$	$\geq$
R	•				•	•
S	•	•				
T	•		•	•	•	•
C					•	•
A	•				•	•

partial tot equiv rel

D if R equiv **For $x \in A$**

"**equiv class**" of x

$$[x] = \{y \in A : xRy\}$$

→ subset of A $(x \subseteq A)$ ... R

" set of equivalence classes" or "quotient set"

$$A/R = \{[x] \mid x \in A\}$$

serves to stick one unique label to items that satisfy the same prop. and to confuse them with the same label / knowing what we do with this label)

EZ $\mathbb{Z}$ remains of ÷ of number by 2

→ 0 or 1

0 equiv class is then "set even"

2 partitions

$0+0=0$ $0+1=0$ $1+1=0$

even even even ...

$0\times0=0$ $0\times1=0$ 1×1 1

ERST congruence

D $f : A \longrightarrow A/R$

$x \longmapsto [x]$

canonical projection

any elt $z \in [x]$ is : "class representative of $[x]$

T E Gr between the set of equiv R on E on all partitions of E

nothing more but a partition on E

Prop

Ex "n divises m"

$n \equiv m \pmod{3}$

$n \mathcal{R} m$

$+ \times \mathbb{Z}$

$\mathbb{Z}/2\mathbb{Z}$ ring

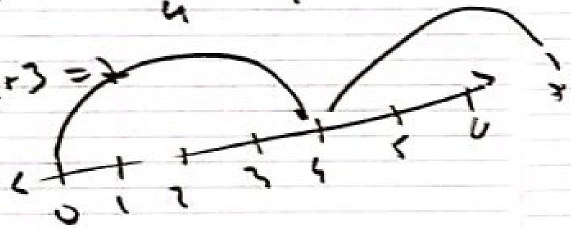
<u>Fundamental arit laws</u>

$+$ prod

<u>Addition</u> sum, 1st, 2nd, terms of the addition

$4 + 3$

$0+4+3 = 4+3 = 7$



P1 Com $A+B = B+A$

P2 Ass $(A+B)+C = A+(B+C)$

P3 $A+0 = 0$ "opposite"

neutral $A + \overline{A} = 0$

$+$ uniqueness

$n+0=n$

$\forall n \in \mathbb{N} \quad n+0=n$

$n+s(y) = s(n+y)$

$s(n) = n+1$

$x_1, x_2, \dots x_n$ any nbs

$x_1+x_2+\dots+x_n = \sum_{i=1}^{n} x_i$

$\sum k x_i = k \sum x_i$

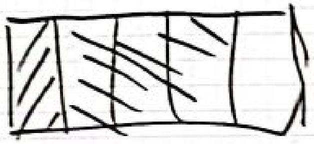
$\sum k = nk$

$\sum (x_i+y_i) = \sum x_i + \sum y_i$

$$\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array} \qquad \begin{array}{r} 1014 \\ +\ 3 \\ \hline 1017 \end{array}$$

$$\begin{array}{r} 9246 \\ +\ 3475 \\ \hline ? \end{array}$$

$$\begin{array}{r} {}^{+1}\ {}^{+1} \\ 9246 \\ +\ 3475 \\ \hline 12719 \end{array}$$



$\frac{1}{5} + \frac{3}{4} = \frac{4}{?}$

seek common fraction equiv

cd

$\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

$\frac{1}{2} + \frac{1}{5} = \frac{5}{5} \cdot \frac{1}{2} + \frac{2}{2} \cdot \frac{1}{5}$

$= \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

$\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1.25$

5/4 [illegible]

Subtraction —

N possible only if $A > B$

$A - B = C$

must satis

$A = B + C$

P1 ~~C..~~

P2 ~~SA..~~

P3 0 only neutral on R

$0 - 5 \neq 0$

P4 opposite

$$\begin{array}{r} {}^{-1} \\ 704 \\ -\ 512 \\ \hline 192 \end{array}$$

704 ← minuend Minuend

512 ← subtrahend

192 ← rest or diff

$$\begin{array}{r} 14 \\ -5 \\ \hline 9 \end{array}$$

$$\begin{array}{r} {}^{-1}\ {}^{-1}\ {}^{-1} \\ 4574 \\ -\ 3785 \\ \hline 0789 \end{array}$$

$$\begin{array}{r} 16 \\ -9 \\ \hline 7 \end{array}$$

Multiplication

multiplier multiplicand Product

m M P

$m \times M = \underset{(1)}{M} + \underset{(2)}{M} + \underset{(3)}{M} + \dots + \underset{(m)}{M} = \sum_{i=1}^{m} M = P$

product factors

$a \times b = a.b = ab$

Peano

$n.0 = 0$

$p(q+1) = pq + q$

power spec not spec as x

power exponent

$n.n.n.n.n.n.n.n = n^{8}$

= "exponentations"

Cuquet 1484

$n^x n^y = n^{x+y}$

$a^x b^x = (ab)^x$

P
COM
ASSOC
unit 1
inverse
distributive oper^n

$a.(b+c) = ab + ac$

reverse = factorize^n

$\prod_{i=1}^{n} x_i$ $\prod k x_i = k^n \prod x_i$

$\prod k = k^n$ $\prod_{i=1}^{n} (x+y) = (x+y)^n$

factorial $1 \times 2 \times 3 \times 4 \times \dots \times n = n!$

$\boxed{0! = 1}$

$$\begin{array}{r} 1014 \\ \times \quad 3 \\ \hline 3042 \end{array}$$

56
3

$$\begin{array}{r} 4574 \\ \times \quad 8 \\ \hline 32 \\ 56 \\ 40 \\ 32 \\ \hline 36592 \end{array}$$

Division

dividend D
divider d
quotient Q — int / eucli / "ovymance" / fraction
rest R — Euclidean division

$R \neq \emptyset$

$$\boxed{D = Q \cdot d \cdot R}$$

$$D : d = \frac{D}{d}$$

$$= D \cdot \frac{1}{d} = D \cdot i_D$$

inverse of d

$$\frac{x}{x} = x^1 \cdot x^{-1} = x^{1-1} = x^0 = 1$$

part inverse or reciprocal

n°1 $\overline{O}$ singularity result undetermined!

2 $\cancel{D} \cdot d$ Dividend × divisor

equiv frac

n3 frac > 0 < 1 "proper fraction"

≠ improper

$$\frac{x \cdot x \cdot x}{y \cdot y} = x^3 \cdot y^{-2}$$

$$\frac{x^n}{x^u} = x^{n-u}$$

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$$

"terms amplifier" "common den"

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

n°1 ~~6/4~~ ~~15/6c~~

unit neutral

sign ≠ ÷ zero conv. err

$$\frac{a}{b} \neq \frac{a+ctc}{b+ctc}$$

$$\frac{a^n}{a^u} = a^{n-u}$$

$$\frac{1}{a^u} = a^{-u}$$

n-root

$$2^3 2^2 = 2^{3+2}$$

$$2^1 = 2^{0.5+0.5}$$
$$= 2^{0.5} \cdot 2^{0.5}$$
$$= 2^{1/2} 2^{1/2}$$

any nb?

D

n^{th} root of a where n positive int is r which, when raised to the power n yields x

$$r^n = x$$

By convention we write:

$$r = x^{1/n} := \sqrt[n]{x}$$

radical (under $1/n$), radix (at the root sign)

radical
sign

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b} \Leftrightarrow \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

if $n \in \mathbb{N}$ odd $\quad \sqrt[n]{a^n} = |a|$

even

$x < 0 \quad y = x^{1/n} = \sqrt[n]{x}$

$b^n = x \quad \mathbb{C}$

Finit polynomials

algebraic

value of is equal to the excess of the sum of the terms preceded by + ... −

factoring" or similar or simplify

"resulting product"

$$P_1 \cdot P_2 \cdot P_3 = (a+b+c)(d+e+f)(g+h+i)$$
$$= a(d+e+f)(g+h+i) + $$
$$+b \text{ ———— }$$
$$+c \text{ ———— }$$

$$n = \prod_{i=1}^{n} n_i$$

Absolute Value $\mathbb{R}$

D $|x|$ ~~abs~~ of a real nb x is:

The non negative value of x without regard to its sign

Namely $|x| = x$ for a $+^{ve}$ x

$-x$ ——— - - $-x$ +ve

& $|0| = 0$

its distance from zero

R

formally

$$|x| = \begin{cases} +x & \text{if } x > 0 \\ -x & \text{if } < \\ 0 & \text{—} = \end{cases}$$

At the origine ~~abs~~ was defined as

$|x| = \sqrt{x^2}$

$|x| = \max(-x, x)$

$x \leq |x|$

$|-x| = |x|$

$|x| \leq y \Leftrightarrow -y \leq x \leq y$

$|x| \geq y \Leftrightarrow x \leq -y \vee x \geq y$

Ex $|x-3| \leq 9$

intuitive concept of distance from real nb 3

$[3-9, 3+3] = [-6, 12]$

useful to interpret $|x-y| = \sqrt{(x-y)^2}$

as euc dist between // on real line

$\mathbb{R}$ to // dist $\to$ metric space

P1 $|x| \geq 0$

$|x| = 0 \Leftrightarrow x = 0$

$|xy| = |x||y|$

Sub +ᵛ $|x+y| \leq |x| + |y|$

$|(|x|)| = |x|$

$|-x| = |x|$

$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

$|x-y| \geq |(|x| - y)|$

"Signum"

$$\operatorname{sgn}(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{—} \; = 0 \\ +1 & \text{—} \; > \end{cases}$$

$$= \frac{d}{dx}|x| \qquad x \neq 0$$

$$x = \operatorname{sgn}(x) \cdot |x|$$

~~Calcul~~ Calculation Rules (operators priorités)

( )

[ ]

{ }

$\{[(\,)]\}$

$$-a \cdot \frac{(b+c)^d}{e^f} - g$$

$-a*(b+c)$^d/e^$f-g$

order prioritization

1 — Negation

2 ^

3 * and / higher?!

4 \ integer division (specific to Gaussian)

5 mod modulo

6 +, −

$= \beta * \alpha$^d/e^$f - g$

P ( )

E x^n

M ×

D ÷

A +

S −

& before

= <> ...

left most higher priority

1 $\neg$
2 $\wedge$
3 $\vee$
4 $\oplus$
5 $\Leftrightarrow$
6 $\Rightarrow$