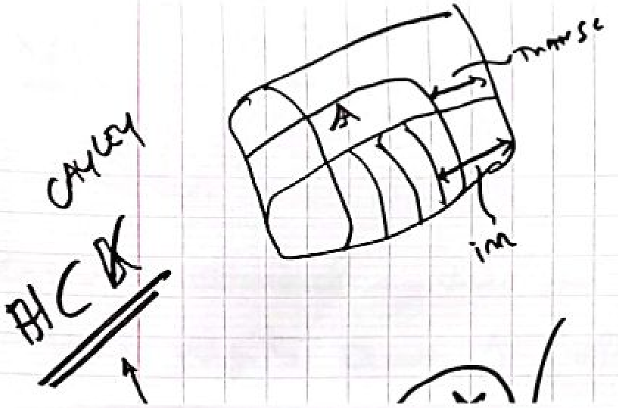
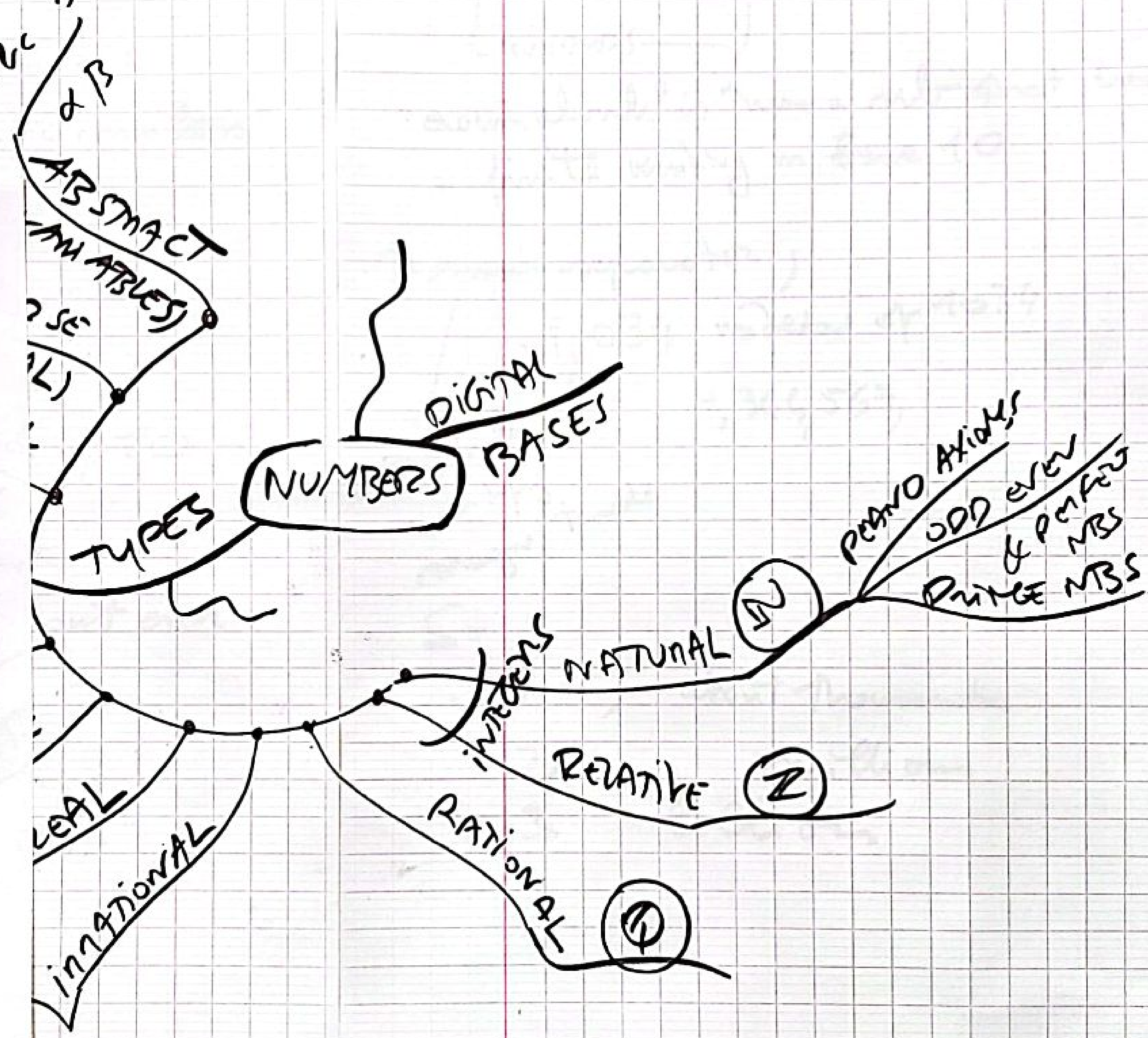
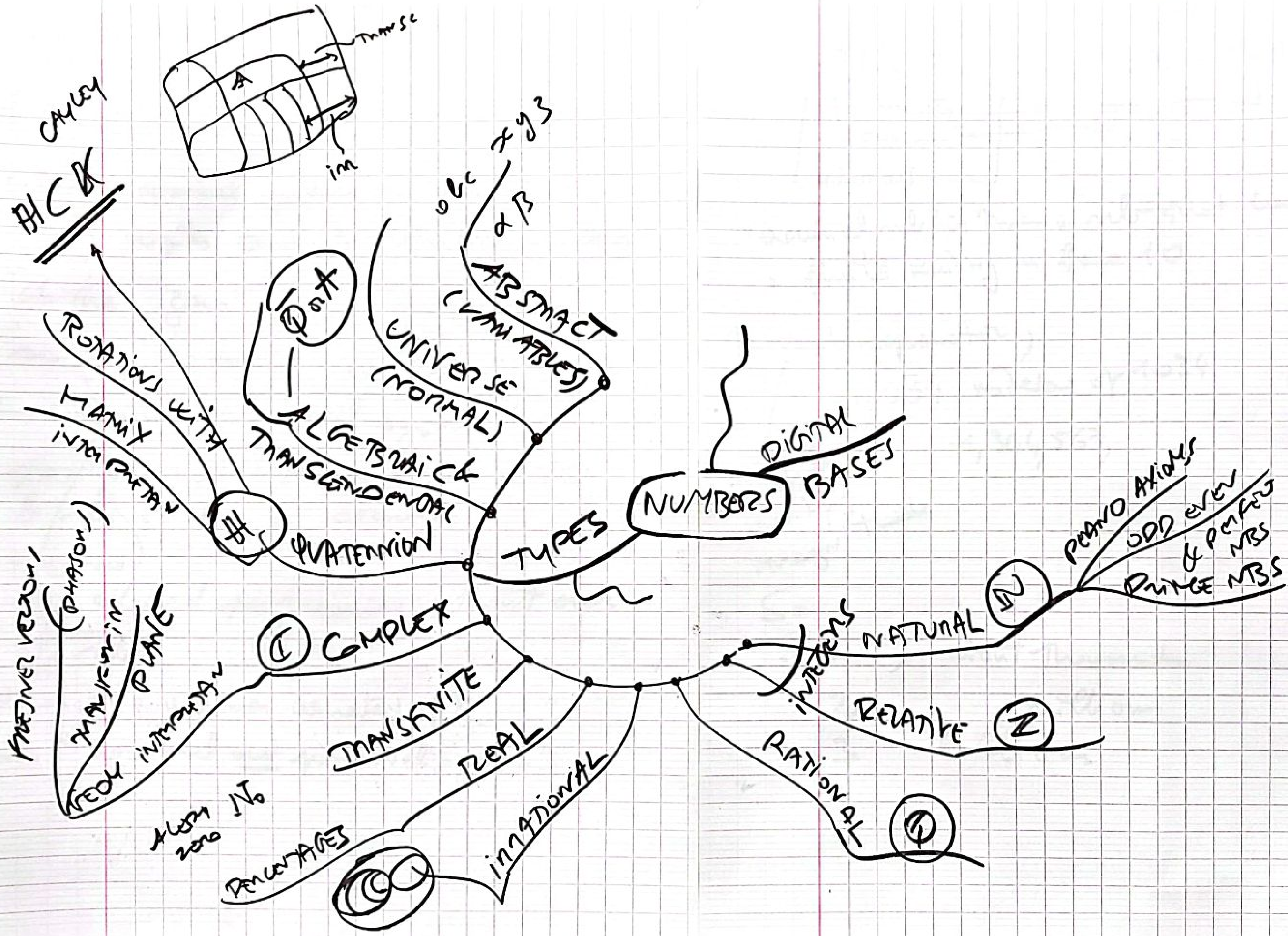


29 p



isoZ NUMBERS


NUMBERS
TYPES
DIGITAL
BASES
NATURAL
PEANO AXIOMS
ODD EVEN & PERFECT NBS
PRIME NBS
INTEGERS
RELATIVE
RATIONAL
IRRATIONAL
REAL
PERCENTAGES
TRANSFINITE
COMPLEX
GEOM INTERPRETATION
PLANE
QUATERNION
MATRIX
INTERPRETATION
ROTATIONS WITH
ALGEBRAIC &
TRANSCENDENTAL
UNIVERSE
(NORMAL)
ABSTRACT
(VARIABLES)
α β
x y z
CAYLEY
Transc
im

NUMBERS

"Scalar"

current decimal system on base 10
uses digits 0 → 9 "Arabic numbers"
"chiffre" "zero"
"nothing" "no amount" ie "zero"
Indians → "positional system"
position digit expresses power of 10
& nb of time it occurs
absence
→ allowed proof reasoning without error
of nbs
absence a a power denoted 0
→ "decimal and positional system"

$$324 = 3 \times 100 + 2 \times 10 + 4 \times 1$$

10^2, 10^1, 10^0

units
tens
hundreds

"decimal nb" is thus a nb that has a finite writing in base 10

thousands separator ,
1,034 instead of 1034
1,344,567,
enriches quality magnitude

So :
- one , about thousands
- 2 millions
- 3 billions

BILLIONS GIGA 10^9
HUNDREDS OF MILLIONS
TENS ___________
MILLIONS MEGA 10^6
HUNDREDS OF THOUSANDS
TENS OF THOUSANDS
THOUSANDS 10^3
HUNDREDS
TENS
UNITS
.
TENTHS
HUNDREDTHS

milli 10^{-3}
micro 10^{-6}

- any integer other than unit can be taken as basis numbering sys
  binary
- generalization
· Any positive integer N can be represented in base b as a sum

where each coef a_i are multiplied by their respective weight b^i

$$N = a_{n-1}b^{n-1} + a_{n-2}b^{n-2} + \dots + a_1 b^1 + a_0 b^0$$
$$= \sum_{i=0}^{n-1} a_i b^i$$

with $a_i \in [0, b-1]$
$b_i \in [1, b^{n-1}]$ any positive integer value 1, 2, 3

R1 base per letters

R2 2 for b maximum value of N will be $2^n - 1$
"Mersenne nbs" can be prime if and only if n is also a prime nb

$b = 10$ $n = 3$
largest $(b-1)10^2 + (b-1)10^1 + (b-1)10^0$
$= 900 + 90 + 9 = 999 = 10^3 - 1$
$= b^3 - 1$

R3 "palindrome"

DIGITAL BASES

b b characters for representing b first nbs

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

"digits"

LEFT

rep order unit immediately above

or b times larger

nb digit ~~to~~ written

D #20

spoken

~~sig~~ single unit "ten"

"hundred"

unit up 1, 2 ~ 3 rule "thousand"

10, 11, 19   ~~18~~ 10 ten a

5b6a71c

R1 rules dec

R2 all + or × of 2 numbers of single digits

R3 decimal

E1 base ten we have seen above

that   142,713

$$142{,}718_{10} = 1.10^5 + 4.10^4 + 2.10^3 + 7.10^2 + 1.10^1 + 3.10^0$$

E2 nb $0110_2 = 0.2^3 + 1.2^2 + 1.2^1 + 0.2^0 = 6_{10}$

TYPE of numbers

Count measure label

Natural integer numbers

- "equality"
- "one" or "unit" "1"

D "equal" if each of the units of one we can match a unique of the other and vice versa (bijective)
if not "inequality"

- "natural sequence $\mathbb{N}$" whole nbs

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$\mathbb{N}^* = \mathbb{N}^1 = \{1, 2, 3, \dots\}$$

R debatable since neither positive nor negative

$\mathbb{Z}^+$ $\mathbb{N} \cup \{0\} = \mathbb{Z}_{\geq 0}$ or $\mathbb{N}_0$

calculative set
or ~~#~~ "whole numbers"

(Frege)

P1 0 is the nb of elts (defined as an equiv R)
of all ~~#~~ sets equiv to (in bij to) the empty set

P2 1 —— only

P3 2 ~~a set~~ is the nb of elts of all sets equiv to the set whose only elt are 1 and 2

P4 in general...

- construc° countable pty
- origin of maths
- seq unlimited but countable
  n $n+1$

D ~~Z~~ two integers that differs from a single positive unit are said to ~~#~~ be "consecutive"

PEANO AXIOMS and Dedekind

$<$ $=$

"0"

s successor

$1 = s(0)$

$2 = \cancel{s(1)}\ s(s(0)) = s(1)$

A1 0 is a natural nb (this permits $\cancel{\mathbb{N}}$ $\mathbb{N}$ to be not empty)

A2 Every ——— has a suc, $s(n)$

therefore s is an injective app
~~to~~ that is to say

$\cancel{\forall x s}$ $\cancel{\forall y}$ $\forall x, y \quad s(x) = s(y)$
$\iff x = y$

A3 suc of a nat nb is never zero
(therefore $\mathbb{N}$ has a first elt)

$\forall x \quad \neg(s(x) = 0)$

A4 If we prove a property φ that is T for x and its successor $s(x)$

then this property is T

for any x ("Recurrence")

$\mathbb{N} = \{0, 1, 2, 3, \ldots, n, \ldots\}$

R also $+ \times$

Odd, Even & Perfect nbs

study parity = determine if is or not a multiple of 2

An integer multiple of 2 is an even integer, the others are odd integers

D_1 0, 2, 4, 6, 8 "even"

n^{th} $E : 2n = n + n \quad \forall \mathbb{N}$

$E = \{x \in \mathbb{N} : \exists n \in \mathbb{N}\ (x = 2n)\}$

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D2   1, 3, 5, 7   odd

$(n+1)^{th}$ $\mathbb{O}: 2n+1 = \overline{E} \quad \forall \mathbb{N}$

$\mathbb{O}: \{x \in \mathbb{N} : \exists n \in \mathbb{N} / (x = 2n+1)$

R "perfect number"
= equal to the sum of their integer divisors strictly smaller than themselves

$6 = 1+2+3$

$28+1 = 2+4+7+14$

Prime numbers

D with exactly 2 positive divisors (both: "1" and the nb itself)
case where there are more than two divisors "Composite nb"   P = "primality"

$\mathbb{P}_{R}$ $\not\ast$

∞ ? yes

Proof
Suppose finite nb
p_1, p_2, p_n
create new nb
$N = (p_1 p_2 \cdots p_n) + 1$
should be divisible by one $\exists$ prime p_i
of the
$N = q \cdot p_i$
... contradiction

$n_1 \leq n$

n^{th} prime number

n^2 ϕ function indicator

Relative integer nbs

. $\mathbb{N}$ issues

$-$

$\div$ partial-result $\not\in \mathbb{Z}$

not an internal operation of $\mathbb{N}$

. $\mathbb{Z} = \dots, -3, -2, -1, 0, 1, 2, 3 \dots$

$\mathbb{N} \subset \mathbb{Z}$

$\mathbb{Z}^+ = \mathbb{Z}_{>0} = \{n \in \mathbb{Z} \mid n > 0\} = \mathbb{N}^*$

$\mathbb{Z}_0^+ = \mathbb{Z}_{\geq 0} =$ ——— $\geq$ $\quad \mathbb{N}$

$\mathbb{Z}_0^-$ $\quad <0$ ——— <0

$\mathbb{Z}^{\neq}$ $\quad \neq 0$ ——— $\neq$

→ Group relative to $+$

. D A "countable set", if it is equipotent to $\mathbb{N}$

= there is a bij of S on $\mathbb{N}$

2 equipot sets have the same nb of elts in the meaning of their cardinal or at least the same ∞^Y

$\mathbb{N}$ and $\mathbb{Z}$ are countable

Proof: let us show that $\mathbb{Z}$ is countable by writing

$x_{2k} = k$ and $x_{2k+1} = -k-1$

for any int $k \geq 0$

$0, -1, 1, -2, 2, -3, 3$

of all rel int only QED

Rational numbers

÷ fraction — rat / mixt ∄

not i.o. of $\mathbb{Z}$

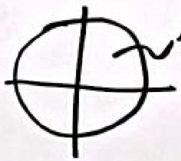
$\frac{2}{3}$ ← numerator dividend

↖ denominator divider

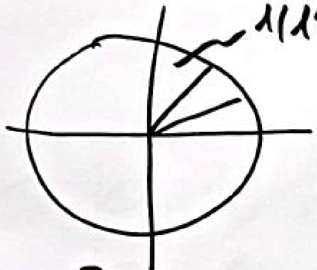
"fractional nb"

part of something / object ...

pizza



1/4



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Set of —

$\mathbb{Q} = \left\{ \frac{p}{q} \mid (p,q) \in \mathbb{Z}, q \neq 0 \right\}$

maybe $= \frac{1}{}$ every integer is a rat nb

and erroneous repetition of above below mean

$R = \left\{ \frac{p}{q} \mid (p,q) \in \mathbb{Z}, q > 0 \text{ and } \gcd(p,q) = 1 \right\}$

$\frac{3}{6}$ $\frac{1}{2}$ no unique form

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

$\mathbb{Z}$ group $\times \div$

Contrary to intuition set $\mathbb{N}$ equip $\mathbb{Q}$!

1/1 → 2/1

3/1 → 1/1

1/2

cantor diagonal met

$f : \mathbb{N} \to \mathbb{Q}$ inj surj

- on the set $\mathbb{Z} \times \mathbb{Z}\{0\}$

$(a,b) \mathcal{R} (a',b') \iff ab' = a'b$

equivalence on $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$

$\mathbb{Q} = (\mathbb{Z} \times \mathbb{Z} \setminus \{0\}) / \mathcal{R}$

equiv class $(a,b) \in \mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ is explicitely denoted by $\frac{a}{b}$

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$$\frac{a}{b} \cdot \frac{p}{q} = \frac{a \cdot p}{b \cdot q} \qquad \frac{a}{b} + \frac{p}{q} = \frac{aq + bp}{bq}$$

$\mathbb{Q}$ Field (Grp commutatif)

$\frac{0}{1}$ neutral $+$

$\frac{1}{1}$ ______ $\times$

$\to$ any non zero elt is reversible

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ab} = \frac{1}{1}$$

$(ab, ab)\, \mathcal{R}\, (1,1)$ $\quad (1,2) \not\mathcal{R} (2,4)$

$\mathcal{R} \to$ introduce equiv $\mathcal{R}$

$$\frac{a}{b} = \frac{a'}{b'} \iff ab' = a'b$$

$\frac{1}{2} = \frac{2}{4}$ because $(1,2)\, \mathcal{R}\, (2,4)$

partial quantities

[illegible]

Irrational numbers

· order discovering

Ex $\sqrt{2}$

suppose it is a rat root

So

$\frac{a}{b}$

3 remaining possibilities

1 a is odd (then b is even)

2 ~~X~~

3 ———— odd

By squaring we have $\sqrt{2} = \frac{a}{b}$

$$2b^2 = a^2$$

...

E2 $e = 1 + \frac{1}{1!} + \frac{1}{2!} \quad \frac{1}{n!}$

If e rat

$q! \times$

$\nearrow \quad q > 1$

contradiction

· $\sqrt{2}$ e

→ must complemented

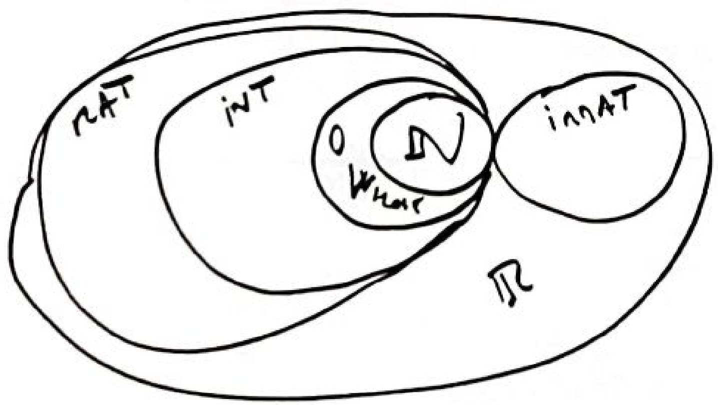
D → any ~~rat~~ real nb that cannot be expressed as a ratio of integers

cannot be represented as terminating or repeating decimals.

Real nbs

$\cup$ rat & irrat $\quad \mathbb{Q} \subset \mathbb{R}$

$\mathbb{R}$ define
properties of topology (among oth)
& especially Cauchy ~~seq~~ seq
"Completed ordered field"



$\mathbb{R}$ countable?

Proof $\quad \mathbb{Q} \xleftrightarrow[bij]{} \mathbb{R}$

$[0,1[$ is then not countable
...
"Cantor diagonal process"
"power of the continuum"

· any real can be approx as close by a rat
(for irrat simply stop ~~at~~ & find corr^{ing} rat

$\mathbb{Q}$ is "dense" in $\mathbb{R}$ $\quad \boxed{\overline{\mathbb{Q}} = \mathbb{R}}$

real x
for any $n \in \mathbb{N}$
$$p_n = [10^n x]$$
the integer part of x that was previously x^{ed} by 10^n

cor Any real nb is a lim of a seq of rat

Percentages

D1 scalar $x \in \mathbb{R}$ expressed in "percentage" =

$$x\% = x \cdot 100$$

per thousand

$$x‰ = x \cdot 1{,}000$$

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Transfinite numbers

card of this set was a nb that existed as such, without that we need to use the symbol ∞

$\aleph_0$ $\qquad$ ~~$\aleph$~~ $\aleph_0 = \mathrm{Card}(\mathbb{N})$

"aleph zero" "transfinite nb"

. decisive act is to assert that there is, after the finite, a transfinite = unlimited scale of determined modes which by nature are infinite, and yet can be specified, as for the finite by spec nbs, well def & distinguishable from each oth!

nec. a a set card can be equal to one of its parts!

. calculus rules

2 infinite sets are ~~...~~ equiv if

there exists a bij

. → show ∞ even nbs is equiv int

~~card(int) = card(even)!~~

. → α_0 2 sets are equipot

a set can be equal to one of its parts

equipot Any One set of all sets ∞!

→ ~~Axiomatic~~ ZF

$$\begin{aligned} \aleph_0 + 1 &= \aleph_0 \\ \aleph_0 + \aleph_0 &= \aleph_0 \\ \aleph_0^2 &= \aleph_0 \end{aligned}$$

abolition as card of the disjoint $\cup$ of count. sets

E1

$\aleph_0$ card($\mathbb{N}$)

$\aleph_0 + \aleph_0$ which is equiv to saying

~~So~~ that we sum the card of $\mathbb{N}$ disjoint

$\cup$ $\mathbb{N}$

But as $\mathbb{N}$ disjoint $\cup$ $\mathbb{N}$ is equipot to

$\mathbb{N}$ then $\aleph_0 + \aleph_0 = \aleph_0$

odd / even) both count & which disj $\cup$ is also count

E2 $\aleph_0 + 1$ ($+$ point) still equipot to $\mathbb{N}$

$\longrightarrow \aleph_0 + 1 = \aleph_0$

$$\text{Card}(\mathbb{N}\times\mathbb{N}) = \text{Car}(\mathbb{N}^2) = [\text{Car}(\mathbb{N})]^2$$

$\longrightarrow$ $\aleph_0^2 = \aleph_0$

$\mathbb{N}$ $\mathbb{Z} = \mathbb{Z}^+ \cup \mathbb{Z}^-$

$\aleph_0 + \aleph_0 = \aleph_0$

$\forall$ = in $\mathbb{Q} \to \mathbb{Z} \times \mathbb{Z}$

$\aleph_0 \times \aleph_0 = (\aleph_0)^2 = \aleph_0$

- an more interesting stat:

Consider card of the set of all the card

it is nec greater than all the card

including itself)

= card of the set of all sets A is greater

than car A itself

$\to$ there is no set containing all sets

since there is a bigger one (with card)

Considering a non empty set A & then

to state that Card(A) $\leq$ Card $\mathcal{P}(A)$

not prove any $f: A \longrightarrow \mathcal{P}(A)$

R

Complex nbs

"imaginary"

- solve pbs with no solutions $\mathbb{R}$
& formalize certain transformation in plan
- Construct
Set theory (x, y) or aof operation
define pure unit imaginary

D1 "unit pure imaginary nb"

$i^2 = -1 \iff i = \sqrt{-1}$

D2 $z = a + \underbrace{ib}_{\text{imaginary}}$

D3 $\mathbb{R} \subset \mathbb{C}$

R $\mathbb{C}$ identified to oriented Euclidian plane E thanks to choice direct orthonormal basis
"Argand Cauchy plane"

Gauss

$x_1 \; x_2$
$y_1 \; y_2$

field

$\mathbb{C} = \{ z = (x + iy) \mid x, y \in \mathbb{R} \}$

$\mathbb{R}[i]$

$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$\cdot \quad x_1 x_2 - y_1 y_2 \quad x_1 y_2 + x_2 y_1$

$\Re(z) = x$

$\Im(z) =$

conjugate $\bar{z} = \overline{x + iy} = x - iy$

z^*

$\Re(z) = \frac{z - \bar{z}}{2}$ $\quad \Im(z) = \frac{z - \bar{z}}{2i}$

"module"
"norm" = lengths from the C of

prop T

$|z| = \sqrt{x^2 + y^2} = \sqrt{z \cdot \bar{z}}$

always ≥ 0

$\sum$ to N of
properties
• DISTANCE

$|z|$ coincides with the absolute value of z when z is real

$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2} = \frac{x_1+iy_1}{x_2+iy_2} \cdot \frac{x_2-iy_2}{x_2-iy_2}$$

$$= \frac{(x_1x_2+y_1y_2) - i(x_1y_2 - x_2y_1)}{x_2^2+y_2^2}$$

$\frac{1}{x+iy}$ → list $\underline{z}$ with prop module & conj

P1 $|z| = 0 \Leftrightarrow z = 0$

2 $|z| = |-z| = |\bar{z}|$

3 $|\mathcal{R}(z)| \leq |z|$ with = if z real

$|\mathcal{I}(z)| \leq |z|$ —— — imag

4 $\forall z_1, z_2 \in \mathbb{C} \times \mathbb{C}$ $|z_1 z_2| = |z_1||z_2|$

if $z_2 \neq 0$ $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{z_2}$

5 $|z|^2 = z \cdot \bar{z}$

6 $z\, z'$ $\bar{\bar{z}} = z$ $\overline{z+z'} = \bar{z} + \bar{z}'$

$\overline{z z'} = \bar{z}\bar{z}'$

R1 Conjugation "involution"

R2 group automorphism

R3 field —— $(\mathbb{C}, +, \times)$

7 $\overline{\left(\frac{1}{z}\right)} = \frac{1}{\bar{z}}$ $\overline{\left(\frac{z}{z'}\right)} = \frac{\bar{z}}{\bar{z}'}$

8 $|z_1 + z_2| \leq |z_1| + |z_2|$

more gen form "Minkowski inequality")

→ calculus

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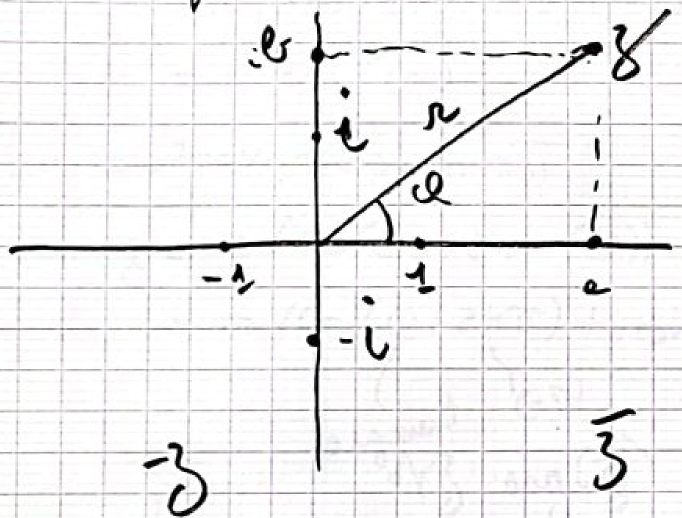
geom interpretation Cmbs

Gauss = plan map

- affix (a,b) $a+ib$



$Aff(\vec{r}) = a+ib$

$$z = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = a+ib$$

canonical base $\left\{ 1 = \begin{pmatrix}1\\0\end{pmatrix}, i = \begin{pmatrix}0\\1\end{pmatrix} \right\}$

$$r = |z| = \sqrt{a^2+b^2}$$

$\begin{pmatrix}1\\0\end{pmatrix}$ unitary basis $\vec{v}$ of unicd by hoz

$\begin{pmatrix}0\\1\end{pmatrix}$ vert $\mathbb{R}i$ module norm ≥ 0

canonical vectors basis of $\mathbb{R}^2$

$$\vec{v} = x\vec{e_1} + y\vec{e_2} = x\begin{pmatrix}1\\0\end{pmatrix} + y\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$\| \ \| = \sqrt{x^2+y^2}$

can identify $\mathbb{C}$ plan with euc plane

$$\left| \frac{a+bi}{b+a} \right| = 1$$

$a = r\cos(\theta)$ $b = r\sin(\theta)$

$r = \sqrt{a^2+b^2}$

$\theta^{-1} = \cos^{-1}\left(\frac{a}{r}\right) = \sin^{-1}\left(\frac{b}{r}\right) = \tan^{-1} b/a$

$\longrightarrow$

$$z = a + ib$$
$$= r\cos(\varphi + ir\sin(\varphi))$$
$$= r(\cos(\varphi) + i\sin(\varphi))$$
$$= r\,\mathrm{cis}(\varphi)$$

= ... modulo 2π

$$z = r(\cos(\varphi) + i\sin(\varphi))$$
$$= r(\cos(\underbrace{\varphi}_{\text{argument of } z \ \arg(z)} + \underbrace{2k\pi}_{\in \mathbb{N}}) + i\sin(\varphi + 2k\pi))$$

$$\arg(\bar{z}) = -\arg(z)$$
$$-(-z) = \arg(z + \pi)$$

Taylor series

$$\cos(\varphi) = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} + \dots + (-1)^k \frac{\varphi^{2k}}{(2k)!} + \dots$$

sin $1 + \dots$

which sum is similar to

$$e^{\varphi} = 1 + \varphi + \frac{\varphi^2}{2!} + \frac{\varphi^3}{3!} + \dots + \frac{\varphi^k}{k!} + \dots$$

but instead perfectly similar to Taylor expression of e^{ix}

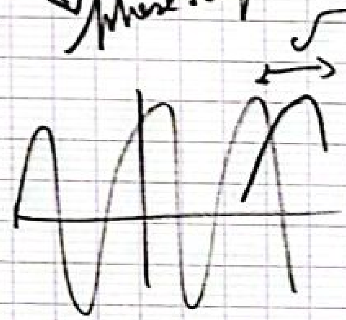
$$e^{i\varphi} = 1 + i\varphi - \frac{\varphi^2}{2!} - i\frac{\varphi^3}{3!} + \dots + i^k\frac{\varphi^k}{k!} + \dots$$
$$= \cos(\varphi) + i\sin(\varphi)$$

$$\underline{z = r(\cos(\varphi) + i\sin(\varphi)) = re^{i\varphi}}$$

"Euler formula"

$$z = r(\cos(\varphi + \delta) + i\sin(\varphi + \delta))$$
$$= re^{i(\varphi + \delta)}$$
$$= re^{i\varphi}\underbrace{e^{i\delta}}_{\text{phase shift factor}}$$

phase shift



$$\cos(\theta) + i\sin(\theta) = e^{i\theta}$$
$$- \quad = e^{-i\theta}$$

"Euler formulas" "se trouve..."

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin \quad \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

can be a purely $\mathbb{C}$ nb

→ all generality trig f can be considered as f that go from $\mathbb{C} \to \mathbb{C}$

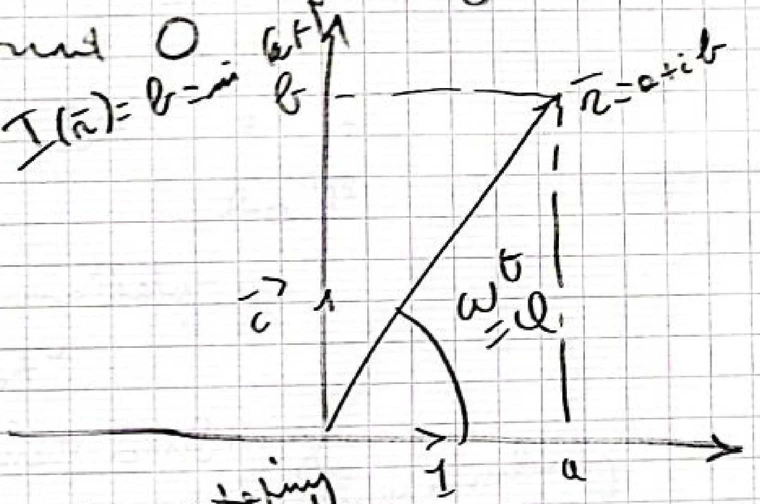
R linearize $\cos^k(\theta)$

$$\sin^6(\theta) = \left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^6$$

... :) lecture!

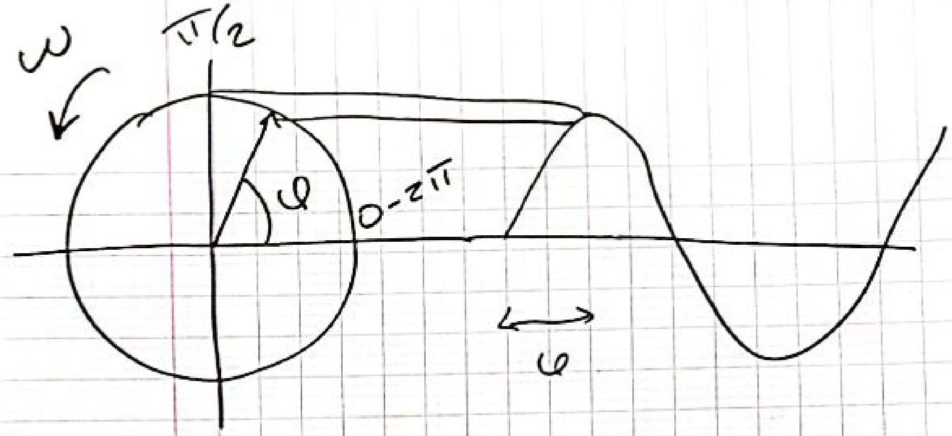
Fresnel vectors (phasors)

$f(t) = r\sin(\omega t)$ can be represented as the projection on vert y axis (imaginary axis) the set (I) of a rotating ~~vector~~ vector $\vec{r}$ at any velocity ω around O



Fresnel diff — vector rotating imaginary part of

$$\vec{r} = R(\vec{r}) + i\,I(\vec{r})$$
$$= r(\cos(\omega t) + i\sin(\omega t)) =$$
$$re^{i\omega t} = \cancel{re^{i\omega t}}\, re^{i\varphi}$$



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Transformation in the plane

$+ \to$ translation

$\times \to$ contracts scaling

"Square root of a transformation"

$$z_1 = x_1 + iy_1$$
$$= \alpha e^{i\alpha}$$

$$z_2 \qquad ? \qquad ?$$
$$\beta e^{i\beta}$$

1 homothety

2 rotation

3 direct similarity

retrograde linear similarity

4

5

6

Quaternion

"hyper $\mathbb{C}$"

D $(a,b,c,d) \in \mathbb{R}^4$ $\mathbb{H}$

"$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

$+ (a',b',c',d') = (a+a', b+b', c+c', d+d')$

R natural addition in $\mathbb{R}^4$ seen a

R vector space

associativity

$$\_ \cdot \_ = \begin{pmatrix} aa' - bb' - cc' - dd' \\ (ab' + ba') + (cd' - dc') \\ (ac' + ca') - (bd' - db') \\ (ad' + da' + (bc' - cb') \end{pmatrix}$$

$\cancel{C}$ $(0,1,0,0) \cdot (0,0,1,0) = (0,0,0,1)$

$\times$ $= (0,0,0,-1)$

$= -1 (0,0,1,0) \cdot (0,1,0,0)$

Nat X ——

X ass with +

Land R — ssim X com

X neutral $(1,0,0,0) \cdot (a,b,c,d)$

$\times$ $= (a,b,c,d)$

any elt $(a,b,c,d) \in \mathbb{H} = \mathbb{H} - \{(0,0,0,0)\}$

non null

$a^2 + b^2 + c^2 + d^2 \neq 0$

C subfield of $(\mathbb{H}, +, \times)$

...

Matrix interpretation of $\mathbb{H}$

Rotations with $\mathbb{H}$

Spinors $\}$ ☆

$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{K}$

~~order~~ Com / Cayley

~~assoc~~

Algebraic & Transcendental nbs

D → "integer of degree n"

any complex that is a sol of an
univariate alg eq degree n
i.e a polynomial —
whose coef are integers, & whose
dominant coef is equal to 1

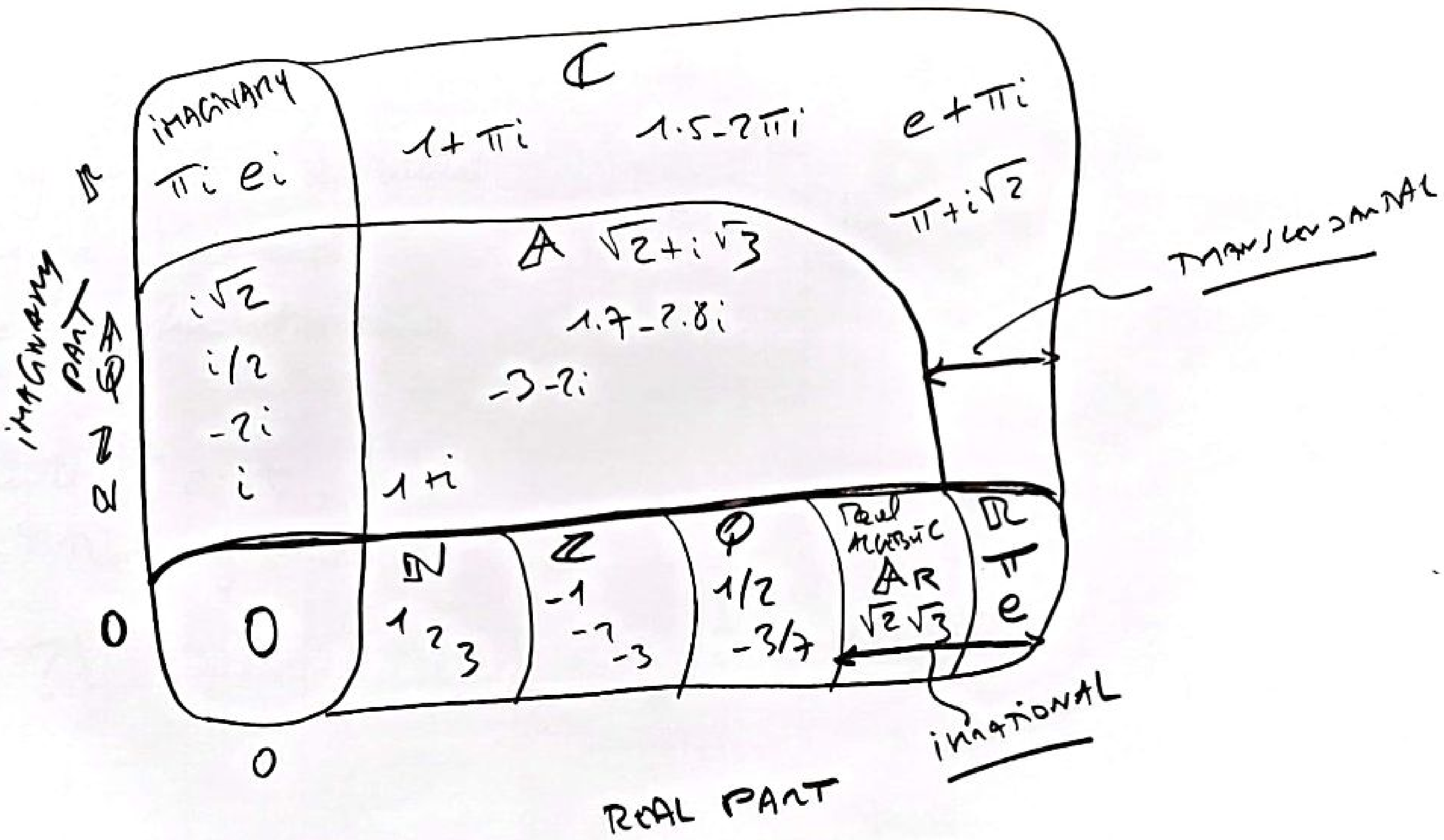
nb — coef net

$\overline{\mathbb{Q}}$ or $\mathbb{A}$

$\mathbb{Z}$ fully closed

real or compl not alg
not root of nonzero poly.
uncount

π e


ℂ
IMAGINARY
πi ei
$1+\pi i$
$1.5-2\pi i$
$e+\pi i$
$\pi+i\sqrt{2}$
TRANSCENDANTAL
𝔸 $\sqrt{2}+i\sqrt{3}$
$1.7-2.8i$
$-3-2i$
$1+i$
$i\sqrt{2}$
$i/2$
$-2i$
i
IMAGINARY PART
ℝ
𝔸
ℚ
ℤ
ℕ
0
ℕ
1 2 3
ℤ
-1 -2 -3
ℚ
$1/2$ $-3/7$
Real ALGEBRIC
$\mathbb{A}_\mathbb{R}$
$\sqrt{2}$ $\sqrt{3}$
ℝ
π e
IRRATIONAL
0
REAL PART

Universe nbs (normal nbs)

D real nb whose infinite seq of digits in every base b is distributed uniformly in the sense that each of the b digit values has the same density 1/b

→ no digit or (finite) [digits, occurs more freq^y that any other.

U

?!

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Abstract nbs (variables)

D nb → as doing abstraction from nature of the ~~[illegible]~~ objects that constitute the group .. cosify

is a nb that does not assign a qty of any particular kind of thing

R 0, 1, 2

↓ n work because they represent only spec cases.

"literal relations" or "unknown"

—— calculus

1 $\underbrace{a, b, c}_{\text{abstract ct}}$ $\underbrace{x, y, z}_{\text{entities variables or ?}}$ CD — metrics or random var

Α α ∂
Β β
Γ γ
Δ δ
Ε ε
Ζ ζ
Η η
Θ θ
Ι ι
Φ φ
Ψ ψ
Λ λ
Μ μ
Ν ν
Ξ ξ
Ο ο
Π π
Ρ ρ
Σ σ
Τ τ Χ
Χ Ω ω

Domain of a variable

$a < b$

D1 "Domain of definition" of a var, all numerical values it is likely to take ~~to~~ between 2 spec lim (end point)

like $\mathbb{N}$ $\mathbb{R}$ $\mathbb{R}^+$

D2 Closed int with end points a and b

$$[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

int notn — set builder notn

open] [— < <

closed

L open right

semi closed L

$]-\infty, b]$ unbounded on b — int closed (or closed R)

$[a, b)$

[

$(-\infty, b]$

isomorus

x "ordered var" if by representing its domain of def by a hor axis where each point on the axis rep a value of x, then for each pair of values we can say there is an "antecedent" one one that is a "subsequent"

"increasing" if each sub value is greater than each ant value

"with monotonic variations"

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